The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 20, 2001 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of ≠ or in radical form. [60]

1 Express 300° in radian measure.

2 If $f(x) = \sqrt{29 - x^2}$, find $f(-2)$.

3 In $\triangle ABC$, $\sin A = \frac{1}{4}$, $\sin B = \frac{1}{8}$, and $b = 20$. What is the length of a?

4 If the number 0.00416 is expressed in scientific notation as 4.16×10^x, what is the value of x?

5 If $f(x) = 2x + 4$ and $g(x) = x^2 + 1$, find $(f \circ g)(3)$.

6 In which quadrant will the image of $A(4,-2)$ lie after dilation D_{-2}?

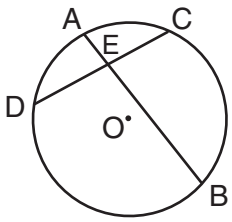
7 Solve for x: $2^{4x-1} = 4^x$

8 In $\triangle ABC$, $m\angle C = 30$ and $a = 24$. If the area of the triangle is 42, what is the length of side b?

9 What is the value of x in the equation $3^x = 148$, expressed to the *nearest hundredth*?

10 Solve for all values of x: $|3x + 5| = 7$

11 In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E, $m\overset{\frown}{AC} = 50$, and $m\overset{\frown}{BD} = 150$. Find $m\angle AED$.

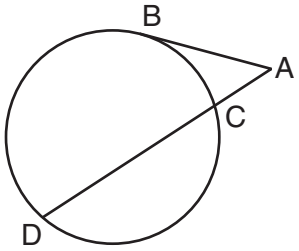


12 In $\triangle ABC$, $a = 6$, $b = 10$, and $m\angle C = 120$. What is the length of c?

13 The probability of Rick getting an A on any test is $\frac{2}{3}$. Find the probability that he earns an A on *exactly* 3 of 4 tests.

14 Evaluate: $\sum_{k=0}^{3}(3k-2)^2$

15 In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to a circle. If $AC = 4$ and $CD = 12$, find AB.



Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 What is the image of point (2,4) under the translation $T_{-6,1}$?

(1) (–4,3)
(2) (–4,5)
(3) (8,3)
(4) (8,5)

17 The expression $3\sqrt{-18} + 5\sqrt{-12}$ is equivalent to

(1) $9i\sqrt{2} + 10i\sqrt{3}$
(2) $6i\sqrt{2} + 7i\sqrt{3}$
(3) $19i\sqrt{5}$
(4) $-90\sqrt{6}$

18 The fraction $\frac{3 - x}{2x - 6}$, $x \neq 3$, is equivalent to

(1) $\frac{1}{2}$ (3) $\frac{1}{4}$

(2) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

19 If x is a real number, what is the solution set of the equation $\sqrt{1 - 2x} = 2$?

(1) $\left\{\frac{3}{2}\right\}$ (3) $\{-2\}$

(2) $\left\{-\frac{3}{2}\right\}$ (4) $\{\ \}$

20 The value of $\sin \frac{4\pi}{3}$ is

(1) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$

(2) $-\frac{1}{2}$ (4) $-\frac{\sqrt{3}}{2}$

21 The expression $\sin^2 x + \cos^2 x - b^2$ is equivalent to

(1) 1 (3) $(1 + b)(1 - b)$

(2) b^2 (4) $\sin x \cos x - b$

22 What is the solution set of the inequality $x^2 - x - 6 < 0$?

(1) $-2 < x < 3$ (3) $x < -2$ or $x > 3$

(2) $-3 < x < 2$ (4) $x < -3$ or $x > 2$

23 In a circle, a central angle whose measure is $\frac{\pi}{2}$ radians intercepts an arc whose length is $\frac{3\pi}{2}$ centimeters. How many centimeters are in the radius of the circle?

(1) 1 (3) 3

(2) 2 (4) 4

24 The graph of the equation $y = 4^{-x}$ lies in Quadrants

(1) I and II (3) III and IV

(2) II and III (4) I and IV

25 On a standardized test with a normal distribution, the mean is 88. If the standard deviation is 4, the percentage of grades that would be expected to lie between 80 and 96 is closest to

(1) 5 (3) 68

(2) 34 (4) 95

26 What is the inverse of the function $y - 2 = 7x$?

(1) $y = \frac{2 - x}{7}$ (3) $y = 7x - 2$

(2) $y = \frac{2x}{7}$ (4) $y = \frac{x - 2}{7}$

27 What is the maximum number of distinct triangles that can be formed if $m\angle A = 30$, $b = 8$, and $a = 5$?

(1) 1 (3) 3

(2) 2 (4) 0

28 The fraction $\frac{b + \frac{b}{a}}{a - \frac{1}{a}}$ is equivalent to

(1) b (3) $\frac{2ab}{a^2 - 1}$

(2) $\frac{b}{a - 1}$ (4) $\frac{a - 1}{b}$

29 As angle θ increases from π radians to 2π radians, the cosine of θ

(1) increases throughout the interval

(2) decreases throughout the interval

(3) increases, then decreases

(4) decreases, then increases

30 The graph of which equation forms an ellipse?

(1) $x^2 - y^2 = 9$ (3) $2x^2 + y^2 = 8$

(2) $2x^2 + 2y^2 = 8$ (4) $xy = -8$

31 Which term describes the roots of the equation $2x^2 + 3x - 1 = 0$?

(1) rational (3) equal

(2) irrational (4) imaginary

32 What is the value of the expression $2x^{-\frac{1}{3}}$ when $x = 8$?

(1) 1 (3) $\frac{1}{2}$

(2) 2 (4) $\frac{1}{4}$

33 What are the sum (S) and product (P) of the roots of the equation $3x^2 - 7x + 12 = 0$?

(1) $S = 7, P = 12$

(2) $S = \frac{7}{3}, P = -4$

(3) $S = \frac{7}{3}, P = 4$

(4) $S = -\frac{7}{3}, P = -4$

34 In simplest form, what is the third term in the expansion of $(x - 2y)^6$?

(1) $-80x^3y^3$ (3) $-60x^4y^2$

(2) $80x^3y^3$ (4) $60x^4y^2$

35 In the set of real numbers, what is the domain of $f(x) = \sqrt{x + 5}$?

(1) $x \geq -5$ (3) $x > -5$

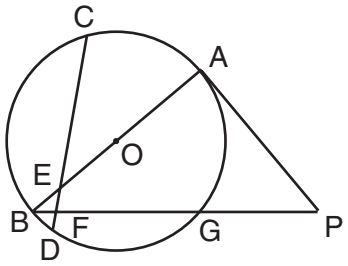
(2) $x \leq -5$ (4) $x \geq 0$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 In the accompanying diagram of circle O, tangent $\overline{PA}$, secant $\overline{PGFB}$, diameter $\overline{AOEB}$, and chord $\overline{CEFD}$ are drawn; $m\overset{\frown}{CA} = 70$; $m\overset{\frown}{DG} = 90$; and $m\angle CEA = 40$.



Find:

a $m\overset{\frown}{CB}$ [2]

b $m\overset{\frown}{BD}$ [2]

c $m\angle APB$ [2]

d $m\angle PAB$ [2]

e $m\angle ABG$ [2]

37 *a* On the same set of axes, sketch and label the graphs of the equations $y = \sin 2x$ and $y = 3\cos x$ in the interval $-\pi \leq x \leq \pi$. [8]

b Based on the graphs drawn in part *a*, find all values of x in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $\sin 2x = 3\cos x$. [2]

38 *a* Express the roots of the equation $9x^2 = 2(3x - 1)$ in simplest $a + bi$ form. [5]

b Solve for x:

$$\frac{12}{x^2 - 16} - \frac{24}{x - 4} = 3 \quad [5]$$

39 Find all values of θ in the interval $0 \leq \theta < 360°$ that satisfy the equation $\sin\theta = 2 + 3\cos 2\theta$. Express your answer to the *nearest ten minutes* or *nearest tenth of a degree.* [10]

40 *a* Peter (P) and Jamie (J) have computer factories that are 132 miles apart. They both ship their completed computer parts to Diane (D). Diane is 72 miles from Peter and 84 miles from Jamie. Using points D, J, and P to form a triangle, find $m\angle PDJ$ to the *nearest ten minutes* or *nearest tenth of a degree.* [6]

b If $\log 2 = a$ and $\log 13 = b$, express in terms of a and b:

(1) $\log 26$ [1]

(2) $\log \frac{8}{\sqrt{13}}$ [3]

41 *a* When $\sin x = -\frac{8}{17}$ and x lies in Quadrant III and $\cos y = -\frac{4}{5}$ and y lies in Quadrant II, what is $\cos (x - y)$? [5]

b For all values of x for which the expressions are defined, prove the following is an identity:

$$\frac{\cos 2x}{\sin x} + \sin x = \csc x - \sin x \quad [5]$$

42 *a* Given: $J = -2 + 5i$ and $K = 3 + 2i$

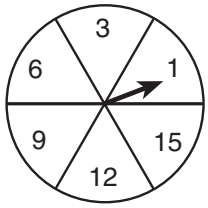
(1) On graph paper, plot and label J and K. [2]

(2) On the same set of axes, plot the sum of J and K and label it L. [1]

(3) On the same set of axes, plot the image of L after a counterclockwise rotation of 270° and label it L'. [2]

(4) Express L' as a complex number. [1]

b The circle in the accompanying diagram is divided into six regions of equal area and has a spinner. The regions are labeled 1, 3, 6, 9, 12, and 15. If the spinner is spun five times, what is the probability that it will land in an even-numbered region *at most* two times? [4]



Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, June 20, 2001 — 9:15 a.m. to 12:15 p.m., only

Part I Score	. . . . .
Part II Score	. . . . .
Total Score	. . . . .
Rater's Initials:	. . . . .

ANSWER SHEET

Pupil . . . . . Sex: ☐ Male ☐ Female Grade . . . . .

Teacher . . . . . School . . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . . .
2	12	22	32 . . . . .
3	13	23	33 . . . . .
4	14	24	34 . . . . .
5	15	25	35 . . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

Tear Here

Tear Here

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, June 20, 2001 — 9:15 a.m. to 12:15 p.m., only

SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) $\frac{5\pi}{3}$

(2) 5

(3) 40

(4) –3

(5) 24

(6) II

(7) $\frac{1}{2}$

(8) 7

(9) 4.55

(10) $-4, \frac{2}{3}$

(11) 80

(12) 14

(13) $\frac{32}{81}$

(14) 70

(15) 8

(16) 2

(17) 1

(18) 2

(19) 2

(20) 4

(21) 3

(22) 1

(23) 3

(24) 1

(25) 4

(26) 4

(27) 2

(28) 2

(29) 1

(30) 3

(31) 2

(32) 1

(33) 3

(34) 4

(35) 1

[OVER]

Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics,* 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* 110 [2]

b 10 [2]

c 50 [2]

d 90 [2]

e 40 [2]

(37) *b* $-\frac{\pi}{2}, \frac{\pi}{2}$ [2]

(38) *a* $\frac{1}{3} \pm \frac{i}{3}$ [5]

b –6, –2 [5]

(39) 56.4°, 123.6°, 270°

or [10]

56°30′, 123°30′, 270°

(40) *a* 115.4° *or* 115°20′ [6]

b (1) $a + b$ [1]

(2) $3a - \frac{1}{2}b$ [3]

(41) *a* $\frac{36}{85}$ [5]

(42) *a* (4) $7 - i$ [1]

b $\frac{192}{243}$ [4]