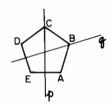
HIGH SCHOOL MATHEMATICS: COURSE III-JANUARY 1981 (1)

Part I

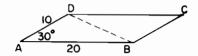
Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

- 1. If m = 8, find the value of $m^{-\frac{2}{3}}$.
- 2. Express in simplest form: $\frac{1 \frac{2}{x}}{\frac{1}{x}}$
- 3. In the accompanying figure, p and q are symmetry lines for regular pentagon *ABCDE*. Find $r_q \circ r_p$ (B).



- 4. Find the value of cos 27° 26' to four decimal places.
- 5. Find the value(s) of θ in the interval $0^{\circ} \le \theta \le 360^{\circ}$ that satisfy the equation $\cos^2 \theta \cos \theta 2 = 0$.
- 6. If x is a positive acute angle and $\sin x = \frac{5}{13}$, find $\cos (2x)$.
- 7. Find: $\sum_{k=1}^{3} k^{k}$
- 8. Solve for x: $81^x = 9^{x+1}$
- 9. Find the value of tan 315°.
- 10. Find the value of cot (Arc $\tan \frac{8}{5}$).
- 11. Chords \overline{AB} and \overline{CD} intersect in a circle at point E. If AE = 6, EB = 3, and ED = 2, find CE.
- 12. From the letters of the word "MATH," a letter is drawn at random and replaced. What is the probability of picking exactly two A's in three tries?
- 13. If a function f is defined by $f(x) = \sin\left(\frac{x}{2}\right)$, find the numerical value of $f(\pi)$.
- 14. Express cos (-175°) as a function of a positive acute angle.
- 15. Express in radian measure an angle of 540°.
- 16. If $\log a = 0.4262$, find $\log \sqrt{a}$.
- 17. In triangle ABC, $a = \sqrt{3}$, $b = \sqrt{3}$, and $m \angle C = 120$. Find the value of c.

18. In the accompanying diagram, ABCD is a parallelogram with AB = 20, AD = 10, and $m \angle A = 30$. Find the area of parallelogram ABCD.



19. Given the following table of SAT scores for a class. What is the range of these scores?

Score	Frequency
210	1
370	5
640	7
700	4
790	1

Directions (20-35): For each question chosen, write in the space provided on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

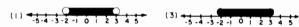
- **20.** If $10^x = m$, then 10^{3x} equals
 - (1) 3m (2) 3+m (3) 3^m (4) m^3
- 21. The expression $\sqrt{-128}$ is equivalent to
 - (1) $2\sqrt{8}$ (2) $-8\sqrt{2}$ (3) $8i\sqrt{2}$ (4) $-8i\sqrt{2}$
- 22. The period of $y = 3 \sin 2x$ is
 - (1) π (2) 2 (3) 3 (4) 4π
- 23. On the same axes, $y = \sin x$ and $y = \cos x$ are graphed. Applying the translation $T_{\frac{\pi}{2},0}$ to $y = \cos x$, the graphs
 - (1) coincide
 - (2) intersect at one point, only
 - (3) intersect at two points, only
 - (4) intersect at three points, only
- 24. The value of $(\sin 13^\circ)(\cos 17^\circ) + (\cos 13^\circ)(\sin 17^\circ)$ equals
 - (1) 1 (2) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 0
- 25. One radian is approximately equal to
 - (1) 57° (2) 45° (3) π , (4) $\frac{\pi}{2}$
- 26. Which geometric abbreviation has vertical line symmetry?
- (1) SSS (2) SAS (3) AAA (4) ASA
- 27. What is the last term in the expansion $(2x 3y)^4$?
 - $(1) -81y^4$ $(2) 81y^4$ $(3) 3y^4$ $(4) -54xy^3$
- 28. The expression $\frac{2}{3+\sqrt{2}}$ is equivalent to
- (1) $6 + 2\sqrt{2}$ (2) $6 2\sqrt{2}$ (3) $\frac{6 + 2\sqrt{2}}{7}$ (4) $\frac{6 2\sqrt{2}}{7}$

HIGH SCHOOL MATHEMATICS: COURSE III-JANUARY 1981 (3)

- 29. The graph of $y = a^x$, where a > 0, contains the point
 - (1) (1,0) (2) (-1,0) (3) (0,0)

- 30. The expression $\cos(2\pi x)$ is equivalent to
 - (1) $-\cos x$ (2) $\cos x$ (3) $-\sin x$

- 31. Which transformation can produce an image triangle with an area not equal to the area of the original triangle?
- (1) translation (2) line reflection (3) dilation (4) rotation
- 32. The roots of $x^2 + bx + 2 = 0$ will be imaginary when b equals (1) 1(2) 7 (3) 3 (4) 9
- 33. Which is the graph of the solution set of $x^2 x 6 < 0$?









34. What is the inverse of the function y - 2 = 3x?

(1)
$$y = \frac{2-3}{3}$$

(2)
$$y = \frac{2}{3}$$

$$(3) y = 3x -$$

(1)
$$y = \frac{2-x}{3}$$
 (2) $y = \frac{2x}{3}$ (3) $y = 3x - 2$ (4) $y = \frac{x-2}{3}$

- 35. The scores on a standardized exam have a mean of 78 and a standard deviation of 3.5. Assuming a normal distribution, a student's score of 86 would rank
 - (1) below the 75th percentile
 - (2) between the 75th and 85th percentile
 - (3) between the 85th and 95th percentile
 - (4) above the 95th percentile

Answers to the following questions are to be written on paper provided by the school.

Part II

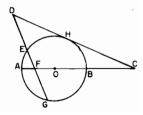
Answer four questions from this part. Show all work unless otherwise directed.

- 36. The heights in inches of a basketball team are as follows: 81, 73, 75, 80, 71. Calculate the standard deviation of these heights to the nearest tenth. [10]
- 37. In the accompanying diagram, \overline{DC} is tangent to circle O at H, \overline{AFOBC} , secant \overline{DEG} intersects secant \overline{CBA} at F, \widehat{mAEH} = 110, $\widehat{\text{m}AG} = 80$, and the ratio between $\widehat{\text{m}AE}$ and $\widehat{\text{m}EH}$ is 2:3.

[1]

Find: $a. \ m\widehat{GB}$

- $b. m\widehat{EA}$
 - [3]
 - c. m∠EFA [3] d. m $\angle DCA$ [3]



HIGH SCHOOL MATHEMATICS: COURSE III-JANUARY 1981 (4)

- 38. a. On the same set of axes, sketch the graphs of $y = \cos 2x$ and $y = \frac{1}{2} \sin x$ for the values of x in the interval $0 \le x \le 2\pi$. [8]
 - b. How many values of x in the interval $0 \le x \le 2\pi$ satisfy the equation $\cos 2x = \frac{1}{2} \sin x$? [2]
- 39. In triangle ABC, $m \angle A = 27^{\circ} 10'$, $m \angle B = 33^{\circ} 20'$, and side BC = 12 centimeters. Find the length of side \overline{AB} to the nearest centimeter. [10]
- 40. a. For all values of x for which the expression is defined, show that the following equality is an identity:

$$2 - \sec^2 x = (\cos 2x) (\sec^2 x)$$
 [4]

- b. Solve for θ in the interval $0^{\circ} \le \theta \le 360^{\circ}$: $\cos 2\theta = -\sin \theta$ [6]
- 41. Triangle ABC has the coordinates A(2, 3), B(9, 1), and C(5, 5).
 - a. (1) On graph paper, draw and label triangle ABC. [1]
 - (2) Find the coordinates of A'B'C', the image of ABC after a reflection through the origin. [3]
 - (3) Find the coordinates of the vertices of triangle A''B''C'', the image of triangle A'B'C' after $r_{y-axis} \circ r_{x-axis} (\Delta A'B'C')$. [4]
 - A series of transformations that reflect point P through the origin, then over the x-axis, and then over the y-axis, is equivalent to a rotation about the origin of

 (1) 90°
 (2) 180°
 (3) 270°
 (4) 360°
 [2]
- 42. a. Using logarithms, find $\sqrt{\pi}$ to the nearest tenth where $\pi = 3.14$. [3]
 - b. Using logarithms, solve for x to the nearest tenth: $2^x = 7$ [4]
 - c. Write, in the form $ax^2 + bx + c = 0$, a quadratic equation whose roots are 3 + i and 3 i. [3]
- 43. A new electronic game has the form of a calculator as shown. There are eight keys: six of them are numbered 1, 2, 3, 4, 5, 6 and two are colored, one red (R) and one yellow (Y). Each key is equally likely to be pressed.



- a. If one key is pressed at random,
 - (1) what is the probability that a number key will be selected? [2]
 - (2) what is the probability that a color key will be selected? [2]
- b. If three keys are pressed at random,
 - (1) what is the probability that exactly two number keys will be selected? [3]
 - (2) what is the probability that at least two number keys will be selected? [3]