The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, August 14, 1997 — 8:30 to 11:30 a.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I
Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of \( \pi \) or in radical form. [60]

1. If \( \star \) is a binary operation defined by \( m \star n = (2m - n)^2 \), find the value of \( 7 \star 4 \).

2. In the accompanying diagram, \( \angle 1 \) and \( \angle 2 \) are supplementary. What is \( m\angle 3 \)?

3. Perform the indicated operation and express in lowest terms:
\[
\frac{x^2 - 25}{14} + \frac{x - 5}{28}
\]

4. The altitudes of two similar triangles are in the ratio 2:3. If the perimeter of the smaller triangle is 18, find the perimeter of the larger triangle.

5. The lengths of the sides of a triangle are 5, 7, and 8. What is the perimeter of the triangle that is formed by joining the midpoints of these sides?

6. If the measure of an exterior angle of one of the base angles in an isosceles triangle is 110°, find the number of degrees in the measure of the vertex angle.

7. What are the coordinates of the image of point (-1,8) after a reflection in the origin?

8. What is the positive root of the equation \( 3x^2 + 5x = 8 \)?

9. In the accompanying diagram of isosceles trapezoid \( ABCD, AB = 18, CD = 10, \) and \( AD = BC = 5 \). Find the height of the trapezoid.

10. Solve the following system of equations for the positive value of \( x \):
\[
\begin{align*}
y &= x \\
y &= x^2
\end{align*}
\]

11. In the accompanying diagram of \( \triangle ABC, \overline{AFB}, \overline{AEC}, \overline{AC} \perp \overline{CB}, \overline{AE} \perp \overline{EF}, \overline{BF} = 8, \overline{FA} = 12, \overline{FE} = 9, \) and \( \overline{BC} = x \). What is the value of \( x \)?

12. The coordinates of \( A \) are \((4,5)\) and the coordinates of \( B \) are \((10,y)\). If the midpoint of \( \overline{AB} \) is \((7,-2)\), find the value of \( y \).

13. A parabola whose equation is \( y = x^2 - 2x + k \) has a turning point with coordinates \((1,-5)\). Find the value of \( k \).
Directions (14–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

14 What is the negation of the statement "The Sun is shining and it is warm"?
(1) The Sun is not shining or it is not warm.
(2) The Sun is shining and it is not warm.
(3) The Sun is shining or it is not warm.
(4) The Sun is not shining or it is warm.

15 What is the sum of \(\frac{x - 2}{3}\) and \(\frac{x - 3}{2}\)?
(1) \(\frac{2x - 5}{2}\)
(2) \(\frac{3x - 5}{3}\)
(3) \(\frac{5x - 13}{6}\)
(4) \(\frac{3x - 5}{2}\)

16 Which equation represents the locus of points equidistant from points (1,1) and (7,1)?
(1) \(x = 4\)
(2) \(y = 4\)
(3) \(x = -4\)
(4) \(y = -4\)

17 In the accompanying diagram, \(\overrightarrow{AG} \parallel \overrightarrow{CED}\), \(m\angle AGF = 30\), and \(m\angle CEF = 45\).

What is \(m\angle GFE\)?
(1) 45
(2) 52
(3) 60
(4) 75

18 After the translation that shifts \((x,y)\) to \((x + 2, y - 2)\), the image of point \(B(-3,0)\) lies in Quadrant
(1) I
(2) II
(3) III
(4) IV

19 If the statements \(a \rightarrow \neg b\) and \(\neg c \rightarrow b\) are true, which statement is a logically valid conclusion?
(1) \(a \rightarrow c\)
(2) \(a \rightarrow \neg c\)
(3) \(b \rightarrow a\)
(4) \(\neg b \rightarrow a\)

20 Which statement about a parallelogram is not always true?
(1) Diagonals are perpendicular.
(2) Opposite sides are congruent.
(3) Opposite angles are congruent.
(4) Consecutive angles are supplementary.

21 Which equation represents the line that passes through the point \((0,1)\) and is parallel to the line whose equation is \(3x + y = 5\)?
(1) \(3x + y = 3\)
(2) \(3x + y = 1\)
(3) \(3x + y = 0\)
(4) \(3x + y = -1\)

22 If the lengths of the diagonals of a rhombus are 6 and 8, the perimeter of the rhombus is
(1) 5
(2) 10
(3) 20
(4) 40

23 Which equation represents a line perpendicular to the line whose equation is \(y = -3x + 2\)?
(1) \(y = -3x - 4\)
(2) \(y = 3x + 4\)
(3) \(y = \frac{1}{3}x - 6\)
(4) \(y = -\frac{1}{3}x - 6\)

24 The vertices of \(\triangle ABC\) are \(A(9,0), B(-3,0), and C(0,5)\). What is the area of \(\triangle ABC\) in square units?
(1) 22.5
(2) 30
(3) 45
(4) 60

25 Which statement is the converse of "If two sides of a triangle are congruent, then the triangle is isosceles"?
(1) If a triangle is not isosceles, then two sides of the triangle are not congruent.
(2) If two sides of a triangle are not congruent, then the triangle is not isosceles.
(3) If a triangle is isosceles, then two sides of the triangle are congruent.
(4) If two sides of a triangle are not congruent, then the triangle is isosceles.

26 If the measure of an exterior angle of a regular polygon is 45°, then the polygon is
(1) a decagon
(2) an octagon
(3) a pentagon
(4) a square
27 What are the coordinates of A', the image of A(1,2) after a reflection in the line y = x?
   (1) (1, -2)                                       (3) (-2, -1)
   (2) (-1, 2)                                       (4) (2, 1)

28 The solution of the quadratic equation
   \( x^2 + 3x - 5 = 0 \) is
   (1) \( \frac{3 \pm \sqrt{11}}{2} \)                  (3) \( \frac{3 \pm \sqrt{29}}{2} \)
   (2) \( -\frac{3 \pm \sqrt{11}}{2} \)               (4) \( -\frac{3 \pm \sqrt{29}}{2} \)

29 Expressed as a fraction in lowest terms,
   \( \frac{x^2 - x - 2}{x^2 - 4}, x \neq \pm 2 \), is equivalent to
   (1) \( \frac{-x - 2}{-4} \)                         (3) \( \frac{x - 1}{x - 2} \)
   (2) \( \frac{x}{x + 2} \)                           (4) \( \frac{x + 1}{x + 2} \)

30 If the lengths of two sides of a triangle measure 7 and 12, the length of the third side could measure
   (1) 16                                          (3) 3
   (2) 19                                          (4) 5

31 If the graphs of the equations \( x^2 + y^2 = 25 \) and \( y = x \) are drawn on the same set of axes, what is the total number of points common to both graphs?
   (1) 1                                         (3) 3
   (2) 2                                         (4) 0

32 A 100-foot wire is extended from the ground to the top of a 60-foot pole, which is perpendicular to the level ground. To the nearest degree, what is the measure of the angle that the wire makes with the ground?
   (1) 31                                         (3) 53
   (2) 37                                         (4) 59

33 How many different six-letter arrangements can be made from the letters in the name "JENNIE"?
   (1) 15                                         (3) 180
   (2) 30                                         (4) 720

34 There are 5 blue pencils and 3 gold pencils in a container. If 2 pencils are drawn at random without replacement, what is the probability that both pencils drawn are gold?
   (1) \( \frac{1}{4} \)                             (3) \( \frac{3}{28} \)
   (2) \( \frac{5}{14} \)                            (4) \( \frac{9}{64} \)

35 Which statement is illustrated in the construction sketched below?

(1) Through a point not on a given line, exactly one line can be drawn perpendicular to the given line.
(2) If two lines cut by a transversal form congruent alternate interior angles, then the two lines are parallel.
(3) If two lines cut by a transversal form congruent corresponding angles, then the two lines are parallel.
(4) If two lines cut by a transversal form same side interior angles that are supplementary, then the two lines are parallel.
Answers to the following questions are to be written on paper provided by the school.

Part II

36 a On graph paper, draw the graph of the equation \( y = x^2 + 4x + 4 \) for all values of \( x \) in the interval \(-5 \leq x \leq 1\). \[6\]

b Describe fully the locus of points 3 units from the turning point of the graph of the equation. \[2\]

c Describe fully the locus of points 5 units from the axis of symmetry of the graph of the equation. \[2\]

37 In the accompanying table, the operation \( \diamond \) is commutative.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>-b</th>
<th>-a</th>
<th>b</th>
</tr>
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<tbody>
<tr>
<td>a</td>
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<td>b</td>
<td></td>
<td>a</td>
<td>-a</td>
<td>b</td>
</tr>
</tbody>
</table>

a On your answer paper, copy and complete the table. \[2\]

b What is the identity element for the operation \( \diamond \)? \[2\]

c What is the inverse of \(-a\) under the operation \( \diamond \)? \[2\]

d Evaluate: \( a \diamond a \diamond a \) \[3\]

e Solve for \( x \): \( -a \diamond a \diamond x = -b \) \[2\]

38 Answer \( a \) and \( b \) for all values of \( x \) for which these expressions are defined.

a Find the values of \( x \) to the nearest hundredth:

\[
\frac{1}{x} = \frac{x + 2}{2x + 3} \quad [4]
\]

b Solve for \( x \) in simplest radical form:

\[
\frac{x^2 + 2x + 4}{x} = \frac{2x}{1} \quad [6]
\]

39 a Given: \((D \land \neg L) \rightarrow R \land \neg R \land D\)

Prove: \( L \) \[5\]

b Three sophomores, four juniors, and two seniors are trying out for the mathematics team. The coach must select three students from this group for the team.

(1) How many selections of three students can be made from this group? \[2\]

(2) What is the probability that one sophomore and two juniors are selected for the team? \[3\]

40 In the accompanying diagram of right triangle \( \triangle LPG \), \( \angle GNP \perp \overline{PL} \), \( LG = 52 \), \( LP = 20 \), and \( \angle NLP = 35^\circ \).

\(\text{G}
\)

\(\text{N}
\)

\(\text{L}
\)

\(\text{P}
\)

\(35^\circ
\)

a Find, to the nearest integer, the length of \( \overline{PN} \). \[2\]

b Find, to the nearest integer, the length of \( \overline{GN} \). \[4\]

c Find, to the nearest degree, the measure of angle \( \angle GLN \). \[4\]

[OVER]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: rectangle $ABCD$ with diagonal $\overline{BFED}$ drawn, $\overline{AE} \perp \overline{BD}$, and $\overline{CF} \perp \overline{BD}$. 

Prove: $\overline{AE} \cong \overline{CF}$ [10]

42 Quadrilateral $ABCD$ has coordinates $A(-2,0)$, $B(6,4)$, $C(2,8)$, and $D(-2,6)$.

Using coordinate geometry, prove that

a) $ABCD$ is a trapezoid [7]

b) $ABCD$ is not an isosceles trapezoid [3]
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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE II

Thursday, August 14, 1997 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Pupil ................................................. Sex: □ Male □ Female Grade .................
Teacher ........................................ School ........................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

1 ........................................ 11 ........................................ 21 ........................................ 31 ........................................
2 ........................................ 12 ........................................ 22 ........................................ 32 ........................................
3 ........................................ 13 ........................................ 23 ........................................ 33 ........................................
4 ........................................ 14 ........................................ 24 ........................................ 34 ........................................
5 ........................................ 15 ........................................ 25 ........................................ 35 ........................................
6 ........................................ 16 ........................................ 26 ........................................
7 ........................................ 17 ........................................ 27 ........................................
8 ........................................ 18 ........................................ 28 ........................................
9 ........................................ 19 ........................................ 29 ........................................
10 .......................................... 20 .......................................... 30 .......................................... 31 ........................................

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

________________________
Signature

Math—Course II—Aug. '97

[7]
FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

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SCORING KEY

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the student’s work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 14–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 100  (11) 15  (21) 2  (31) 2
(2) 35   (12) -9   (22) 3   (32) 2
(3) 2(x + 5) (13) -4   (23) 3   (33) 3
(4) 27   (14) 1    (24) 2   (34) 3
(5) 10   (15) 4    (25) 3   (35) 3
(6) 40   (16) 1    (26) 2
(7) (1, -8) (17) 4    (27) 4
(8) 1    (18) 3    (28) 4
(9) 3    (19) 1    (29) 4
(10) 1   (20) 1    (30) 1

[OVER]
Part II

Please refer to the Department's publication Guide for Rating Regents Examinations in Mathematics, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) \( b \ (x + 2)^2 + y^2 = 9 \)

or

A circle whose center is \((-2, 0)\) and radius is 3.

\( c \ x = 3 \text{ and } x = -7 \)

or

Two lines parallel to the line \(x = -2\) and 5 units from it.

(37) \( b \ b \) [2]

\( c \ a \) [2]

\( d \ -a \) [2]

\( e \ -b \) [2]

(38) \( a \pm 1.73 \) [4]

\( b \ \pm \sqrt{5} \) [6]

(39) \( b \ 18 \) \( \frac{84}{84} \) [3]