The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, August 17, 1989—8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 If \( b \otimes c \) is defined as \( b^2 - c \), find the value of \( 3 \otimes 2 \).

2 Find the value of \((a \otimes d) \otimes (b \otimes c)\) in the system defined below.

\[
\begin{array}{c|cccc}
\otimes & a & b & c & d \\
\hline
a & c & d & a & b \\
b & d & a & b & c \\
c & a & b & c & d \\
d & b & c & d & a \\
\end{array}
\]

3 What is the area of the triangle whose vertices are (2,3), (6,3), and (3,5)?

4 In the accompanying diagram, trapezoid \( ABCD \) is similar to trapezoid \( AFGH \). If \( AF = 18 \), \( AB = 54 \), and \( HG = 9 \), what is the length of \( DE \)?

5 If one root of the equation \( x^2 - 5x + k = 0 \) is 3, find the value of \( k \).

6 In the accompanying diagram of \( \triangle ABC \), \( DE \parallel AB \), \( CA = 9 \), \( DA = 3 \), and \( CE = 10 \). Find \( EB \).

7 In \( \triangle DEF \), \( \angle E = 60 \) and the measure of an exterior angle at \( F \) is 150. Which is the longest side of \( \triangle DEF \)?

8 In the accompanying diagram, \( AB \parallel CE \), \( CF \parallel E \), \( \angle ABC = 50 \), and \( \angle GED = 20 \). Find \( \angle BCF \).

9 Write an equation of the locus of points equidistant from the lines whose equations are \( y = 2 \) and \( y = -4 \).

10 How many different five-letter arrangements can be made from the letters in the name “NANNY”?

11 Given four equations:

\[
\begin{align*}
y &= 2 \\
y &= 0 \\
x &= 4 \\
y &= x^2
\end{align*}
\]

If one of these equations is picked at random, what is the probability that the graph of the equation will pass through the point (0,2)?

12 Find, in terms of \( \pi \), the area of the circle whose equation is \( x^2 + y^2 = 9 \).

13 An equation of the axis of symmetry of the graph of \( y = x^2 + 6x + 9 \) is \( x = -3 \). What is the \( y \)-coordinate of the turning point?
14 The length of the hypotenuse of a right triangle is 12. Find the length of the median to the hypotenuse.

Directions (15–34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

15 Which statement is the negation of \( \sim r \lor t \)?
(1) \( r \lor \sim t \)
(2) \( r \land t \)
(3) \( \sim r \land \sim t \)
(4) \( r \land \sim t \)

16 In the accompanying diagram, \( \triangle ABC \) and \( \triangle ADC \) are isosceles triangles with \( AB \equiv BC \) and \( AD \equiv DC \). Which statement must be true?

\[ \angle 1 \equiv \angle 3 \]
\[ \angle B \equiv \angle D \]
\[ \triangle ABC \sim \triangle ADC \]
\[ \angle 1 \equiv \angle 4 \]

17 One endpoint of a segment has coordinates \((16,3)\). If the coordinates of the midpoint are \((9,6)\), what are the coordinates of the other endpoint?

(1) \((12,5,4,5)\)
(2) \((2,9)\)
(3) \((9,3)\)
(4) \((25,9)\)

18 A quadrilateral whose diagonals do not always bisect each other is

(1) an isosceles trapezoid
(2) a rectangle
(3) a square
(4) a rhombus

19 In the accompanying diagram of \( \triangle ABC \), \( \triangle DEF \) is formed by joining the midpoints of the sides of \( \triangle ABC \). If \( DE = 9 \), \( FE = 18 \), and \( DF = 13 \), what is the perimeter of \( \triangle ABC \)?

\[ 10 \]
\[ 20 \]
\[ 40 \]
\[ 80 \]

20 What is the negation of \( \exists y \neq 2 \)?

(1) \( \forall y = 2 \)
(2) \( \forall y \neq 2 \)
(3) \( \exists y = 2 \)
(4) \( \exists y = 2 \)

21 The slope of the line determined by the points \((-3,2)\) and \((2,-3)\) is

(1) \(-1\)
(2) \(3\)
(3) zero
(4) undefined

22 The hypotenuse of a right triangle has length 15 and one leg has length 5. The length of the other leg is

\[ 10 \]
\[ 10 \sqrt{2} \]
\[ 20 \sqrt{2} \]
\[ 100 \]

23 The measures of the interior angles of a quadrilateral are in the ratio of 1:2:3:4. What is the measure of the smallest exterior angle of the quadrilateral?

(1) 36
(2) 72
(3) 108
(4) 144

24 If the equation of a circle is
\[ (x + 3)^2 + (y - 4)^2 = 36 \], the coordinates of the center are

(1) \((3,-4)\)
(2) \((-3,4)\)
(3) \((-4,3)\)
(4) \((4,-3)\)

(Math. Course II-Aug. '89)
25 Given the true statements: \[ j \rightarrow c \quad \sim c \]
Which conclusion is valid?
(1) \( j \) \hspace{1cm} (3) \( j \wedge c \)
(2) \( \sim j \) \hspace{1cm} (4) \( \sim c \rightarrow j \)

26 Points A and B are 6 units apart. What is the total number of points that are equidistant from A and B and also 3 units from A?
(1) 1 \hspace{1cm} (3) 3
(2) 2 \hspace{1cm} (4) 4

27 In \( \triangle ABC \), the measure of an exterior angle at B is 5 more than twice the measure of \( \angle C \). If \( m \angle C = 65 \), what is \( m \angle A \)?
(1) 45 \hspace{1cm} (3) 110
(2) 70 \hspace{1cm} (4) 135

28 Which set of numbers can not represent the sides of a triangle?
(1) \{2,3,4\} \hspace{1cm} (3) \{3,2,1\}
(2) \{3,4,5\} \hspace{1cm} (4) \{4,3,4\}

29 If the three angles of a triangle are represented by \( (x + 30)^{\circ} \), \( (4x + 30)^{\circ} \), and \( (10x - 30)^{\circ} \), the triangle must be
(1) obtuse \hspace{1cm} (3) right
(2) isosceles \hspace{1cm} (4) scalene

30 Which equation represents a line that is parallel to the line whose equation is \( y = 3x - 1 \)?
(1) \( y = -\frac{1}{3}x + 1 \) \hspace{1cm} (3) \( y = -3x - 1 \)
(2) \( y = \frac{1}{3}x - 1 \) \hspace{1cm} (4) \( y = 3x + 1 \)

31 How many different five-member committees can be formed from a group of eight boys and seven girls if the committee must contain three boys and two girls?
(1) \( _8C_5 \) \hspace{1cm} (3) \( _8P_5 \)
(2) \( 8C_3 \cdot 7C_2 \) \hspace{1cm} (4) \( 8C_3 \cdot 7C_2 \)

32 What is the length of the radius of the circle whose center is the origin and that passes through the point \((-5,-12)\)?
(1) 7 \hspace{1cm} (3) \( \sqrt{17} \)
(2) 13 \hspace{1cm} (4) \( \sqrt{119} \)

33 Which is a point of intersection of the graphs of \( y = 4 - x \) and \( y = x^2 - x^2 \)?
(1) \((3,1)\) \hspace{1cm} (3) \((-2,6)\)
(2) \((-1,5)\) \hspace{1cm} (4) \((4,0)\)

34 Which is an equation whose roots are 3 and \(-2\)?
(1) \( x^2 + x - 6 = 0 \) \hspace{1cm} (3) \( x^2 - 5x - 6 = 0 \)
(2) \( x^2 - x - 6 = 0 \) \hspace{1cm} (4) \( x^2 + 6x - 1 = 0 \)

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct the locus of points equidistant from points A and B.
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed. [30]

36 The accompanying table defines the operation \( \$ \) on the set \( \{E,M,H,S\} \).

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a What is the identity element? [2]
b What is the inverse of \( M \)? [2]
c Does \((E \$ S) \$ H\) equal \(E \$ (S \$ H)\)? [Write yes or no.] [2]
d Solve for \(x\): \(H \$ x = E\) [2]
e Solve for \(y\): \(y \$ y = M\) [2]

37 a Find, in radical form, the roots of the equation \(x^2 - 3x + 1 = 0\). [3]
b On graph paper, draw the graph of the function \(y = x^2 - 3x + 1\), including all values of \(x\) such that \(-2 \leq x \leq 5\). [5]
c Between which two positive, consecutive whole numbers does the larger root of \(x^2 - 3x + 1 = 0\) lie? [2]

38 Find the area of figure \(ABCDE\) with vertices \(A(-3,2), B(3,-2), C(9,5), D(5,5),\) and \(E(1,8)\). [10]

39 In the accompanying diagram of right triangle \(PQR\), \(QS\) is the altitude to hypotenuse \(PR\), \(SR\) is 1 more than \(PS\), and \(PQ\) is 2 more than \(PS\).

a Write an equation which can be used to find \(PS\). [4]
b Find \(PS\). [6]

40 There are four men and three women on a bowling team. For a tournament, five of the seven will bowl and two will serve as alternates.
a If chosen at random, what is the probability that the five-member team will consist of three men and two women? [4]
b What is the probability that the five-member team will have no women? [2]
c If Donna, the best bowler, must be on the team, how many teams of five are possible? [2]
d Once the team of five is chosen, how many ways can they be arranged in the lineup? [2]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41 Given: Quadrilateral $ABCD$ with diagonal $\overline{AE}$ bisecting $\angle DAB$, $\overline{BE}$, $\overline{DE}$, and $\angle 1 \equiv \angle 2$.

Prove: $\overline{BC} \equiv \overline{DC}$

42 Given: If Susan wins the lottery, then she buys a car.
If Susan does not win the lottery, then she will have to walk.
If Susan buys a sailboat and a car, then she is broke.
Susan is not broke.
Susan buys a sailboat.

Let $L$ represent: "Susan wins the lottery."
Let $C$ represent: "Susan buys a car."
Let $W$ represent: "Susan will have to walk."
Let $S$ represent: "Susan buys a sailboat."
Let $B$ represent: "Susan is broke."

Using $L$, $C$, $W$, $S$, and $B$, prove that Susan will have to walk.
ANSWER SHEET

Pupil .................................................. Teacher ..................................................

School .......................................................... Grade ..........................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

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35 Answer question 35 on the other side of this sheet.
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

__________________________
Signature

Math.-Course II-Aug. '89
FOR TEACHERS ONLY
SCORING KEY
THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS
COURSE II

Thursday, August 17, 1989 — 8:30 to 11:30 a.m., only

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the pupil’s work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 7 \hspace{1cm} (11) \ 
\frac{1}{4} \hspace{1cm} (21) 2 \hspace{1cm} (31) 4

(2) a \hspace{1cm} (12) 9\pi \hspace{1cm} (22) 2 \hspace{1cm} (32) 2

(3) 4 \hspace{1cm} (13) 0 \hspace{1cm} (23) 1 \hspace{1cm} (33) 3

(4) 27 \hspace{1cm} (14) 6 \hspace{1cm} (24) 2 \hspace{1cm} (34) 2

(5) 6 \hspace{1cm} (15) 4 \hspace{1cm} (25) 2 \hspace{1cm} (35) construction

(6) 5 \hspace{1cm} (16) 4 \hspace{1cm} (26) 1

(7) \overline{EF} \text{ or } EF \text{ or } d \hspace{1cm} (17) 2 \hspace{1cm} (27) 2

(8) 30 \hspace{1cm} (18) 1 \hspace{1cm} (28) 3

(9) y = -1 \hspace{1cm} (19) 4 \hspace{1cm} (29) 2

(10) 20 \hspace{1cm} (20) 1 \hspace{1cm} (30) 4

[OVER]
Please refer to the Department publication *Guide for Rating Regents Examinations in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36)  $a \; S \quad [2]$

$b \; M \quad [2]$

$c \; \text{Yes} \quad [2]$

$d \; M \quad [2]$

$e \; E \text{ and } H \quad [1,1]$

(37)  $a \; \frac{3 \pm \sqrt{5}}{2} \quad [3]$

$c \; 2 \text{ and } 3 \quad [2]$

(38)  57 \quad [10]$

(39)  $a \; \text{Analysis and equation} \quad [4]$

$b \; 4 \quad [6]$

(40)  $a \; \frac{4}{7} \quad [1]$

$b \; 0 \quad [2]$

$c \; 15 \quad [2]$

$d \; 120 \quad [2]$

Notice . . .

The January 1990 Regents examination in Course II, Three-Year Sequence for High School Mathematics, will be the last examination based on the original 1977 Syllabus. The June 1990 examination will be based on the revised syllabus (1989). If you have not received a copy of the revised syllabus, contact Frederic Paul, Chief, Bureau of Mathematics Education, State Education Department, Albany, NY 12234.