The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, August 14, 1985 – 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

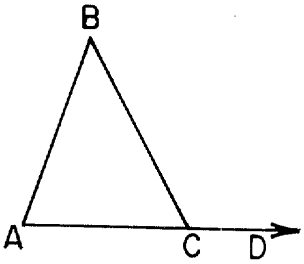
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form. [60]

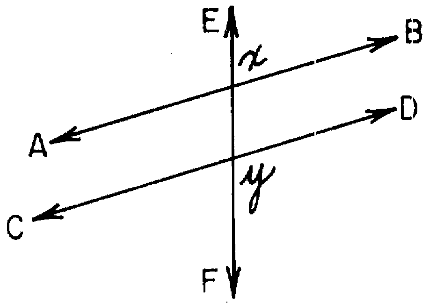
1 Three vertices of rectangle $ABCD$ are $A(0,0)$, $B(5,0)$, and $C(5,2)$. Find the coordinates of D.

2 The lengths of the sides of a triangle are 2, 6, and 7. If the perimeter of a similar triangle is 45, what is the length of its longest side?

3 In the accompanying diagram of $\triangle ABC$, $\overline{AC}$ is extended through C to D. If $m\angle BCD = 120$ and $m\angle A > m\angle B$, which is the longest side of $\triangle ABC$?



4 In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, line EF intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, $m\angle x = a + 10$, and $m\angle y = 2a + 20$. Find the value of a.



5 If @ is a binary operation defined as $x @ y = x^2 - y$, evaluate $3 @ -2$.

6 Write an equation of the line whose y-intercept is -1 and which is parallel to the line whose equation is $y = 3x + 5$.

7 A crew of four must be chosen for the next space shuttle journey. If there are nine qualified astronauts, how many different crews can be chosen?

8 How many different seven-letter permutations can be formed from the letters in the word "BETWEEN"?

9 The table below defines the operation $\oplus$ for the set $S = \{a,b,c,d,e\}$. What is the inverse element of e under $\oplus$?

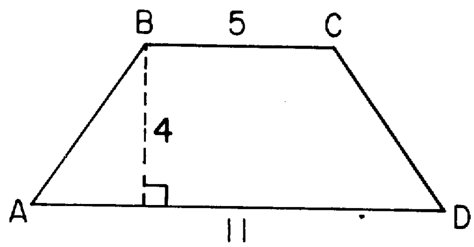
$\oplus$	a	b	c	d	e
a	c	d	e	a	b
b	d	e	c	b	a
c	e	c	a	c	d
d	a	b	c	d	e
e	b	a	d	e	c

10 A set contains four quadrilaterals: a rectangle, a square, a rhombus, and an isosceles trapezoid. If one quadrilateral is selected at random from the set, what is the probability that the figure selected will have congruent opposite angles?

11 In triangle ABC, points X, Y, and Z are the midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. If the perimeter of triangle ABC is 25, find the perimeter of triangle XYZ.

12 In right triangle ABC, $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$. If $CD = 6$, $AD = 4$, and $DB = 3x$, find x.

13 In the accompanying figure, isosceles trapezoid $ABCD$ has bases of lengths 5 and 11 and an altitude of length 4. Find AB.



14 Using the accompanying table, find the value of x in the equation $x \oplus b = a$.

$\oplus$	a	b	c	d
a	c	d	a	b
b	d	c	b	a
c	a	b	c	d
d	b	a	d	c

Directions (15 – 34): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 Which statement is logically equivalent to $\sim(p \vee \sim q)$?

(1) $\sim p \vee \sim q$
(2) $\sim p \vee q$
(3) $p \wedge q$
(4) $\sim p \wedge q$

16 What are the coordinates of the midpoint of a line segment whose endpoints are $(5c,2c)$ and $(3c,0)$?

(1) $(8c^2,2c^2)$
(2) $(8c,2c)$
(3) $(4c,c)$
(4) $(4c^2,0)$

17 Which set of integers can *not* be the lengths of the sides of a triangle?

(1) $\{1,2,3\}$
(2) $\{2,3,4\}$
(3) $\{3,4,5\}$
(4) $\{6,7,8\}$

18 In an isosceles right triangle, the length of the hypotenuse is $2\sqrt{2}$. The length of a leg of the triangle is

(1) 8
(2) 2
(3) 3
(4) 4

19 An equation of the axis of symmetry of the graph of $y = x^2 + 6x - 10$ is

(1) $y = 3$
(2) $y = -3$
(3) $x = 3$
(4) $x = -3$

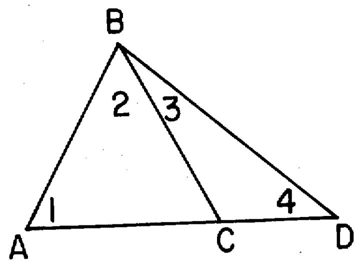
20 An equation of the locus of points 4 units from the origin is

(1) $x^2 + y^2 = 16$
(2) $x^2 + y^2 = 4$
(3) $x = 4$
(4) $y = 4$

21 Points A and D are 5 centimeters apart. The total number of points equidistant from A and D, and also 2 centimeters from A is

(1) 1
(2) 2
(3) 3
(4) 0

22 In the accompanying figure, $\overline{AB} \cong \overline{BC}$ and the points A, C, and D lie on a straight line. Which statement about the indicated angles is *always* true?



(1) $m\angle 1 > m\angle 2$
(2) $m\angle 1 > m\angle 4$
(3) $m\angle 3 > m\angle 4$
(4) $m\angle 4 > m\angle 2$

23 The negation of $\exists_x \sqrt{x^2} = x$ is

(1) $\forall_x \sqrt{x^2} \neq x$
(2) $\exists_x \sqrt{x^2} < x$
(3) $\forall_x \sqrt{x^2} > x$
(4) $\exists_x \sqrt{x^2} \neq x$

24 Which is an equation of a line whose slope is *not* 2?

(1) $y = 2x + 5$
(2) $2y + 1 = 4x$
(3) $3x - 6y = 4$
(4) $5y - 10x + 3 = 0$

25 The roots of the equation $x^2 - 8x + 15 = 0$ are p and q. Which statement best describes them?

(1) $(p > 0) \wedge (q > 0)$
(2) $(p < 0) \wedge (q > 0)$
(3) $(p < 0) \wedge (q < 0)$
(4) $(p = 0) \wedge (q > 0)$

26 The locus of points 3 units from the y-axis which passes through the point $(3,-2)$ is

(1) $x = -2$
(2) $x = 3$
(3) $y = 3$
(4) $y = -2$

27 If two angles of one triangle are congruent to two angles of another triangle, then those triangles *must* be

(1) scalene
(2) isosceles
(3) congruent
(4) similar

28 A circle whose center is (1,5) passes through the point (4,–2). What is the length of the radius of the circle?

(1) $\sqrt{18}$
(2) $\sqrt{45}$
(3) $\sqrt{58}$
(4) $\sqrt{74}$

29 What are the coordinates of the minimum point of the parabola $y = x^2 - 4x + 10$?

(1) (4,10)
(2) (2,6)
(3) (0,10)
(4) (–2,22)

30 The roots of the equation $5x^2 + 7x - 1 = 0$ are

(1) $x = \dfrac{7 \pm \sqrt{69}}{10}$
(2) $x = \dfrac{7 \pm \sqrt{29}}{10}$
(3) $x = \dfrac{-7 \pm \sqrt{69}}{10}$
(4) $x = \dfrac{-7 \pm \sqrt{29}}{10}$

31 Which is logically equivalent to the statement, "Students who master their lessons pass their tests"?

(1) Students who pass their tests master their lessons.
(2) Students who do not master their lessons do not pass their tests.
(3) Students who do not pass their tests do not master their lessons.
(4) Students who do not pass their tests master their lessons.

32 If the slope of one diagonal of a rhombus is 3, then the slope of the other diagonal is

(1) $-\frac{1}{3}$
(2) $\frac{1}{3}$
(3) 3
(4) –3

33 The expression ${}_{25}C_{20}$ is equivalent to

(1) ${}_{25}P_{20}$
(2) ${}_{25}C_{5}$
(3) ${}_{25}P_{5}$
(4) 5!

34 Which set with the indicated operations forms a field?

(1) the set of rational numbers with the operations addition and multiplication
(2) the set of whole numbers with the operations addition and multiplication
(3) the set of real numbers greater than –5 and less than 5 with the operations addition and multiplication
(4) the set of odd integers with the operations addition and multiplication

Directions (35): Leave all construction lines on the answer sheet.

35 *On your answer sheet,* construct the perpendicular bisector of $\overline{AB}$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed. [30]

36 Given $(M_4, +, \bullet)$ where $M_4 = \{0,1,2,3\}$, + is addition clock 4, and $\bullet$ is multiplication clock 4.

a Construct an addition table and a multiplication table for M_4. [4]

b Use the tables constructed in part *a* to answer the following:

(1) What is the additive inverse of 1? [2]

(2) What is the multiplicative inverse of 3? [2]

(3) Solve for x:

$2 \bullet (x + 3) = 2$ [1,1]

37 *a* Draw the graph of $y = 2x^2 + 8x - 3$, including all values of x from $x = -5$ to $x = 1$. [6]

b On the same set of axes, draw the graph of $2x - y = 3$. [2]

c Write the coordinates of the points of intersection of the graphs in parts *a* and *b*. [1,1]

38 Solve the following system of equations algebraically and check:

$$y = 3x^2 - 5x - 2$$
$$y - x = 7$$
[8,2]

39 A box of candy contains 6 mints, 3 chocolates, and 1 lemon drop.

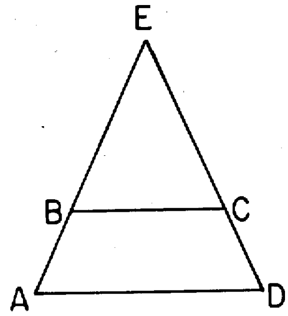
a How many combinations of 3 candies are possible? [2]

b How many combinations of 3 candies will be of one kind only? [3]

c How many combinations of 3 candies will contain exactly one of each kind? [3]

d If 3 candies are selected at random, what is the probability that they will be all of one kind or one of each kind? [2]

40 In the accompanying diagram of trapezoid $ABCD$, $\overline{BC} \parallel \overline{AD}$ and sides $\overline{AB}$ and $\overline{DC}$ are extended to point E forming $\triangle AED$. If $AB = 5$, $BC = 8$, and AD is 2 more than BE, find BE. [*Only an algebraic solution will be accepted.*] [5,5]



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Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41 Given: If laws are good and strictly enforced, then crime will diminish.
If laws are not strictly enforced, then the problem is critical.
Crime has not diminished.
Laws are good.

Let G represent: "Laws are good."
Let S represent: "Laws are strictly enforced."
Let D represent: "Crime has diminished."
Let P represent: "The problem is critical."

Using G, S, D, and P, prove: "The problem is critical." [10]

42 Given: triangle ABC with vertices $A(-2,-1)$, $B(4,7)$, and $C(-4,3)$.

a Prove that $\triangle ABC$ is a right triangle. [5]

b Prove that the median to the hypotenuse is equal to one half of the hypotenuse. [5]

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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH — COURSE II

Wednesday, August 14, 1985 — 8:30 to 11:30 a.m., only

Part I Score	....
Part II Score	....
Part III Score	....
Total Score	....
Rater's Initials:	....

ANSWER SHEET

Pupil .... Teacher ....

School .... Grade ....

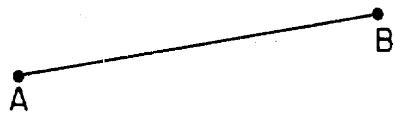
Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 ....
2	12	22	32 ....
3	13	23	33 ....
4	14	24	34 ....
5	15	25	35 **Answer question 35 on the other side of this sheet.**
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

35



Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, August 14, 1985 — 8:30 to 11:30 a.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15 – 34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) (0,2) *or* $\begin{matrix} x = 0 \\ y = 2 \end{matrix}$	(11) $12\frac{1}{2}$	(21) 4	(31) 3
(2) 21	(12) 3	(22) 2	(32) 1
(3) $\overline{BC}$ *or* BC *or* a	(13) 5	(23) 1	(33) 2
(4) 50	(14) d	(24) 3	(34) 1
(5) 11	(15) 4	(25) 1	(35) construction
(6) $y = 3x - 1$	(16) 3	(26) 2	
(7) 126	(17) 1	(27) 4	
(8) 840	(18) 2	(28) 3	
(9) c	(19) 4	(29) 2	
(10) $\frac{3}{4}$	(20) 1	(30) 3	

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a*

$$\begin{array}{c|cccc} + & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 & 0 \\ 2 & 2 & 3 & 0 & 1 \\ 3 & 3 & 0 & 1 & 2 \end{array} \qquad \begin{array}{c|cccc} \bullet & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 \\ 2 & 0 & 2 & 0 & 2 \\ 3 & 0 & 3 & 2 & 1 \end{array}$$

[4]

b (1) 3 [2]
(2) 3 [2]
(3) 0,2 [1,1]

(37) *c* (0,–3) and (–3,–9) [1,1]
or
$x = 0$, $y = -3$ and $x = -3$, $y = -9$

(38) (–1,6) and (3,10) [8]
or
$x = -1$, $y = 6$ and $x = 3$, $y = 10$

(39) *a* 120 [2]
b 21 [3]
c 18 [3]
d $\frac{39}{120}$ [2]

(40) Analysis [5]
10 [5]