The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, June 19, 1997 — 1:15 to 4:15 p.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of \( \pi \) or in radical form.

1. Using the table below, compute \((1 \star 5) \star (2 \star 7)\).

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2. In the accompanying diagram, line \( \ell \) is parallel to line \( k \), line \( m \perp \) line \( k \), and \( m \angle x = m \angle y \). Find \( m \angle x \).

3. If \( \Diamond \) is a binary operation defined as \( a \Diamond b = \sqrt{a^2 + b^2} \), find the value of \( 12 \Diamond 5 \).

4. In the accompanying diagram of similar triangles \( ABE \) and \( ACD \), \( ABC, AED, AB = 6, BC = 3 \), and \( ED = 4 \). Find the length of \( AE \).

5. How many different 5-letter arrangements can be formed from the letters in the word "DANNY"?

6. Evaluate: \( \frac{4!}{3!} \)

7. In the accompanying diagram of \( \triangle ABC \), \( AB \) is extended to \( E \) and \( D \), exterior angle \( CBD \) measures 130°, and \( m \angle C = 75 \). Find \( m \angle CAE \).

8. In right triangle \( ABC \), \( \angle C \) is a right angle and \( m \angle B = 60 \). What is the ratio of \( m \angle A \) to \( m \angle B \)?

9. In \( \triangle ABC \), \( m \angle A = 3x + 40 \), \( m \angle B = 8x + 35 \), and \( m \angle C = 10x \). Which is the longest side of the triangle?

10. A bookshelf contains seven math textbooks and three science textbooks. If two textbooks are drawn at random without replacement, what is the probability both books are science textbooks?

11. Express the product in lowest terms:

\[ \frac{x^2 - x - 6}{3x - 9} \cdot \frac{2}{x + 2} \]

12. In rhombus \( ABCD \), the measure of \( \angle A \) is 30° more than twice the measure of \( \angle B \). Find \( m \angle B \).

13. The endpoints of the diameter of a circle are \((-6,2)\) and \((10,-2)\). What are the coordinates of the center of the circle?

14. Find the area of a triangle whose vertices are \((1,2),(8,2)\), and \((1,6)\).

15. Find the distance between points \((-1,-1)\) and \((2,-5)\).

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16 In the accompanying diagram, the bisectors of \( \angle A \) and \( \angle B \) in acute triangle \( ABC \) meet at \( D \), and \( m \angle ADB = 130 \). Find \( m \angle C \).

\[ \text{Diagram with labeled angles} \]

17 Point \( P \) is on line \( m \). What is the total number of points 3 centimeters from line \( m \) and 5 centimeters from point \( P \)?

18 The diagonals of a rhombus are 8 and 10. Find the measure of a side of the rhombus to the nearest tenth.

Directions (19–34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

19 In isosceles triangle \( ABC \), \( AB = BC \), point \( D \) lies on \( AC \), and \( BD \) is drawn. Which inequality is true?

(1) \( m \angle A > m \angle ADB \)  
(2) \( m \angle C > m \angle CDB \)  
(3) \( BD > AB \)  
(4) \( AB > BD \)

20 If the statements \( m, m \rightarrow p \), and \( r \rightarrow \sim p \) are true, which statement must also be true?

(1) \( \sim r \)  
(2) \( \sim p \)  
(3) \( r \wedge \sim p \)  
(4) \( \sim p \lor \sim m \)

21 If a point in Quadrant IV is reflected in the \( y \)-axis, its image will lie in Quadrant

(1) I  
(2) II  
(3) III  
(4) IV

22 In right triangle \( ABC \), \( m \angle C = 90 \), \( m \angle A = 63 \), and \( AB = 10 \). If \( BC \) is represented by \( a \), then which equation can be used to find \( a \)?

\[ \text{Diagram with labeled angles and sides} \]

(1) \( \sin 63^\circ = \frac{a}{10} \)  
(2) \( a = 10 \cos 63^\circ \)  
(3) \( \tan 63^\circ = \frac{a}{10} \)  
(4) \( a = \tan 27^\circ \)

23 If point \( R'(6,3) \) is the image of point \( R(2,1) \) under a dilation with respect to the origin, what is the constant of the dilation?

(1) 1  
(2) 2  
(3) 3  
(4) 6

24 What is an equation of a line that passes through the point \( (0,3) \) and is perpendicular to the line whose equation is \( y = 2x - 1 \)?

\[ \text{Diagram with axes and lines} \]

(1) \( y = -2x + 3 \)  
(2) \( y = 2x + 3 \)  
(3) \( y = -\frac{1}{2}x + 3 \)  
(4) \( y = \frac{1}{2}x + 3 \)

25 What is an equation of the function shown in the accompanying diagram?

\[ \text{Graph with functions} \]

(1) \( y = x^2 + 3 \)  
(2) \( y = -x^2 + 3 \)  
(3) \( y = -x^2 - 3 \)  
(4) \( y = (x - 3)^2 \)
26 What is an equation of the line that is parallel to the y-axis and passes through the point (2,4)?
(1) \( x = 2 \) \hspace{1cm} (3) \( x = 4 \)
(2) \( y = 2 \) \hspace{1cm} (4) \( y = 4 \)

27 In the accompanying diagram, the altitude to the hypotenuse of right triangle \( ABC \) is 8.

The altitude divides the hypotenuse into segments whose measures may be
(1) 8 and 12 \hspace{1cm} (3) 6 and 10
(2) 3 and 24 \hspace{1cm} (4) 2 and 32

28 If the coordinates of the center of a circle are (-3,1) and the radius is 4, what is an equation of the circle?
(1) \( (x-3)^2 + (y+1)^2 = 4 \)
(2) \( (x+3)^2 + (y-1)^2 = 16 \)
(3) \( (x+3)^2 + (y+1)^2 = 4 \)
(4) \( (x-3)^2 + (y+1)^2 = 16 \)

29 Which expression is a solution for the equation \( 2x^2 - x = 7 \)?
(1) \( \frac{-1 \pm \sqrt{57}}{2} \)
(2) \( \frac{1 \pm \sqrt{57}}{2} \)
(3) \( \frac{-1 \pm \sqrt{57}}{4} \)
(4) \( \frac{1 \pm \sqrt{57}}{4} \)

30 If the complement of \( \angle A \) is greater than the supplement of \( \angle B \), which statement must be true?
(1) \( m\angle A + m\angle B = 180 \)
(2) \( m\angle A + m\angle B = 90 \)
(3) \( m\angle A < m\angle B \)
(4) \( m\angle A > m\angle B \)

31 How many different four-person committees can be formed from a group of six boys and four girls?
(1) \( \frac{10!}{4!} \)
(2) \( 10^4 \)
(3) \( \binom{6}{2} \binom{4}{2} \)
(4) \( \binom{10}{4} \)

32 Which equation represents the axis of symmetry of the graph of the equation \( y = x^2 - 4x - 12 \)?
(1) \( y = 4 \)
(2) \( x = 2 \)
(3) \( y = -2 \)
(4) \( x = -4 \)

33 What is \( \frac{1}{x} + \frac{1}{1-x} \), \( x \neq 1, 0 \), expressed as a single fraction?
(1) \( \frac{x}{x(1-x)} \)
(2) \( \frac{-1}{x(x+1)} \)
(3) \( \frac{2}{-x} \)
(4) \( \frac{1}{x(x-1)} \)

34 In the accompanying diagram, \( RL \perp LF \), \( LR \perp RT \), and \( M \) is the midpoint of \( TP \).

Which method could be used to prove \( \triangle TMR \cong \triangle PML \)?
(1) SAS \cong SAS \hspace{1cm} (3) HL \cong HL
(2) AAS \cong AAS \hspace{1cm} (4) SSS \cong SSS

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct an equilateral triangle in which \( AB \) is one of the sides.
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]

36  a  On graph paper, draw the graph of the equation \( y = x^2 - 8x + 2 \), including all values of \( x \) in the interval \( 0 \leq x \leq 8 \). [6]

b  Find the roots of the equation \( x^2 - 8x + 2 = 0 \) to the nearest hundredth. [Only an algebraic solution will be accepted.] [4]

37  The coordinates of the endpoints of \( \overline{AB} \) are \( A(-2,4) \) and \( B(4,1) \).

a  On a set of axes, graph \( \overline{AB} \). [1]

b  On the same set of axes, graph and state the coordinates of

(1) \( \overline{A'B'} \), the image of \( \overline{AB} \) after a reflection in the \( x \)-axis [2]

(2) \( \overline{A''B''} \), the image of \( \overline{A'B'} \) after a translation that shifts \( (x,y) \) to \( (x + 2,y) \) [2]

c  Using coordinate geometry, determine if \( \overline{A'B'} \equiv \overline{A''B''} \). Justify your answer. [5]

38  Answer both \( a \) and \( b \) for all values for which these expressions are defined.

a  Solve for \( x \): \( -\frac{2}{5} + \frac{x + 4}{x} = 1 \) [4]

b  Express the difference in simplest form:

\[
\frac{3y}{y^2 - 4} - \frac{2}{y + 2}
\] [6]

39  Solve the following system of equations algebraically and check:

\[
\begin{align*}
y &= 2x^2 - 4x - 5 \\
2x + y + 1 &= 0
\end{align*}
\] [8.2]

40  In the accompanying diagram of \( \triangle ABC \), altitude \( AD = 13, \overline{AB} \parallel \overline{AC} \), and \( m\angle BAC = 70 \).

a  Find \( BC \) to the nearest tenth. [4]

b  Using the answer from part \( a \), find, to the nearest tenth, the

(1) area of \( \triangle ABC \) [2]

(2) perimeter of \( \triangle ABC \) [4]

GO RIGHT ON TO THE NEXT PAGE.
Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: If Sue goes out on Friday night and not on Saturday night, then she does not study.
If Sue does not fail mathematics, then she studies.
Sue does not fail mathematics.
If Sue does not go out on Friday night, then she watches a movie.
Sue does not watch a movie.

Let \( A \) represent: "Sue fails mathematics."
Let \( B \) represent: "Sue studies."
Let \( C \) represent: "Sue watches a movie."
Let \( D \) represent: "Sue goes out on Friday night."
Let \( E \) represent: "Sue goes out on Saturday night."

Prove: Sue goes out on Saturday night. [10]

42 Given: parallelogram \( ABCD \), \( DFC \), \( AEB \), \( ED \) bisects \( \angle ADC \), and \( EB \) bisects \( \angle ABC \).

Prove: \( EB \cong DF \) [10]
ANSWER SHEET

Pupil ....................................................... Sex: □ Male □ Female Grade .............
Teacher ................................................... School ............................................

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1 ................. 11 ......................... 21 ........................ 31 .........................
2 ................. 12 ......................... 22 ........................ 32 .........................
3 ................. 13 ......................... 23 ........................ 33 .........................
4 ................. 14 ......................... 24 ........................ 34 .........................
5 ................. 15 ......................... 25 ........................ 35 Answer question 35 on the other side of this sheet.
6 ................. 16 ......................... 26 ........................
7 ................. 17 ......................... 27 ........................
8 ................. 18 ......................... 28 ........................
9 ................. 19 ......................... 29 ........................
10 ............... 20 ......................... 30 ........................
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

___________________________
Signature

Math.-Course II-June '97
FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, June 19, 1997 — 1:15 to 4:15 p.m., only

SCORING KEY

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 19-34, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 2  (11) $\frac{2}{3}$  (21) 3  (31) 4
(2) 45  (12) 50  (22) 1  (32) 2
(3) 13  (13) 2,0  (23) 3  (33) 1
(4) 8  (14) 14  (24) 3  (34) 2
(5) 60  (15) 5  (25) 2  (35) construction
(6) 504  (16) 80  (26) 1
(7) 125  (17) 4  (27) 4
(8) $\frac{1}{2}$  (18) 6.4  (28) 2
(9) $\overline{AC}$  (19) 4  (29) 4
(10) $\frac{1}{15}$  (20) 1  (30) 3

[OVER]
Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) $b$ 0.26 and 7.74  [4]

(37) $b$
  (1) $A'(-2,-4), B'(4,1)$  [2]
  (2) $A''(0,-4), B''(6,-1)$  [2]

(38) $a$ 10  [4]

$b$ $\frac{y + 4}{y^2 - 4}$  [6]

(39) $(2,-5)$ and $(-1,1)$  [8]

Check  [2]

(40) $a$ 18.2  [4]

$b$  
  (1) 118.3  [2]
  (2) 49.9  [4]