The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, June 21, 1988—1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form. [60]

1  The sides of a triangle measure 3, 5, and 7. If the smallest side of a similar triangle measures 9, find its longest side.

2  In the accompanying diagram, \( \overrightarrow{AC} \) and \( \overrightarrow{BDE} \) are parallel. Parallel lines \( AB, CD, \) and \( EF \) are drawn. If \( m \angle 1 = 45 \), find \( m \angle 4 \).

3  If \( \triangle MAR \) is an equilateral triangle, find the measure of an exterior angle at \( R \).

4  An urn contains 3 red marbles and 1 green marble. If 2 marbles are selected at random, without replacement, what is the probability that they are both green?

5  In \( \triangle ABC \), \( m \angle A = 80 \) and \( m \angle B = 50 \). If \( AB = 4x - 4 \) and \( AC = 2x + 16 \), what is the value of \( x \)?

6  In the accompanying diagram of \( \triangle ABC \), \( \overrightarrow{AD} \equiv \overrightarrow{DC}, \overrightarrow{DE} \parallel \overrightarrow{AB} \), and \( DE = 4 \). Find \( AB \).

7  What is the area of the triangle whose vertices are \((0,0), (3,0)\), and \((0,4)\)?

8  If \( s \) is a root of the equation \( x^2 - 4x + k = 0 \), find \( k \).

9  How many different four-digit numerals can be formed from the digits in 1988?

10  The length of the diagonal of a square is 6. Find the length of a side of the square.

11  In right triangle \( ABC \), \( \overrightarrow{CD} \) is the altitude to hypotenuse \( AB \). If \( AD = 2 \) and \( DB = 18 \), find \( CD \).

12  A set contains five quadrilaterals: a parallelogram, a rectangle, a rhombus, a square, and a trapezoid. If one quadrilateral is selected at random from the set, what is the probability that the figure selected will have congruent opposite angles?

13  Solve the following system of equations for the positive value of \( y \):

\[
\begin{align*}
x + y^2 &= 13 \\
x &= -3
\end{align*}
\]

14  Line \( m \) and line \( n \) are parallel and 6 units apart. Point \( Q \) lies on line \( m \). How many points are 3 units from point \( Q \) and equidistant from lines \( m \) and \( n \)?

Directions (15–34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

15  A statement that is logically equivalent to \((a \land b) \land (b \land c)\) is

(1) \( a \land c \)  
(2) \( c \land a \)  
(3) \( a \land \neg b \)  
(4) \( c \land \neg b \)
16. If $\oplus$ is a binary operation defined by $a \oplus b = \frac{a^2 - b^2}{a + b}$, what is the value of $5 \oplus 2$?

(1) 1  
(2) $\frac{6}{7}$  
(3) 3  
(4) $\frac{3}{7}$

17. The statement $\neg (\neg p \land q)$ is logically equivalent to

(1) $p \land \neg q$  
(2) $\neg p \lor \neg q$  
(3) $p \lor \neg q$  
(4) $\neg p \lor \neg q$

18. Which is an equation of a line that is parallel to the line whose equation is $y = 3x + 7$?

(1) $y = -\frac{1}{3}x + 6$  
(2) $y = -3x + 6$  
(3) $y = \frac{1}{3}x - 5$  
(4) $y = 3x - 5$

19. The coordinates of three vertices of parallelogram $ABCD$ are $A(-1,0)$, $B(4,0)$, and $C(5,4)$. What are the coordinates of vertex $D$?

(1) (0.4)  
(2) (1,4)  
(3) (0,3)  
(4) (1,3)

20. Using the accompanying table, which is the solution set for the equation $2 \oplus x = 2$?

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(1) $\{6\}$  
(2) $\{8\}$  
(3) $\{6,8\}$  
(4) $\{4,6\}$

21. Which is an equation of the locus of points 3 units below the line whose equation is $y = 2x$?

(1) $x = -1$  
(2) $y = -1$  
(3) $x = 5$  
(4) $y = 5$

22. Under which operation is the set $\{1,2,4,8,16,\ldots\}$ closed?

(1) addition  
(2) subtraction  
(3) multiplication  
(4) division

23. What are the roots of the equation $ax^2 + bx + c = 0$?

(1) $x = \frac{-b \pm \sqrt{b^2 - 2ac}}{4a}$  
(2) $x = \frac{b + \sqrt{b^2 - 4ac}}{2a}$  
(3) $x = \frac{-b + \sqrt{b^2 + 4ac}}{2a}$  
(4) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

24. If the coordinates of $A$ are $(3,4)$ and the coordinates of $B$ are $(-3,-4)$, then the length of $AB$ is

(1) 5  
(2) 10  
(3) 20  
(4) 100

25. Given the true statements:

$p \lor q$

$p \Rightarrow r$

$\neg q$

Which statement must also be true?

(1) $q$  
(2) $r$  
(3) $\neg p$  
(4) $p \land q$

26. If $M(-2,5)$ is the midpoint of $AB$ and the coordinates of $A$ are $(4,7)$, what are the coordinates of $B$?

(1) $(1,6)$  
(2) $(2,12)$  
(3) $(-8,6)$  
(4) $(-8,3)$

27. What is the $y$-intercept of the graph of the equation $y = x^2 - 2x + 3$?

(1) 1  
(2) 2  
(3) 3  
(4) $-2$

28. If $nC_2 = 45$, what is the value of $n$?

(1) 5  
(2) 8  
(3) 9  
(4) 10

29. Which statement is always true?

(1) A square is a rhombus.
(2) A parallelogram is a square.
(3) A rhombus is a rectangle.
(4) A rectangle is a rhombus.
30 What are the coordinates of the center of the circle whose equation is \((x - 3)^2 + (y + 5)^2 = 16\)?

(1) (0,4)  
(2) (4,0)  
(3) (-3,5)  
(4) (3,-5)

31 In \(\triangle ABC\), \(\angle C\) is a right angle. If the slope of \(AC\) is \(\frac{2}{3}\), then the slope of \(BC\) equals

(1) \(\frac{2}{3}\)  
(2) \(-\frac{2}{3}\)  
(3) \(-\frac{3}{2}\)  
(4) \(\frac{3}{2}\)

32 If the measures of the angles of a triangle are represented by \(x + 30\), \(4x + 30\), and \(10x - 30\), the triangle must be

(1) isosceles  
(2) obtuse  
(3) right  
(4) scalene

33 If the lengths of two sides of a triangle are 6 and 8, the length of the third side may be

(1) 7  
(2) 2  
(3) 14  
(4) 15

34 Which is an equation of the axis of symmetry of the graph whose equation is \(y = x^2 + 8x - 10\)?

(1) \(y = -4\)  
(2) \(y = 4\)  
(3) \(x = -4\)  
(4) \(x = 4\)

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, in parallelogram \(ABCD\), locate by construction the point on side \(DC\) that is equidistant from points \(A\) and \(B\).
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed. [30]

36 Given: \( \{r, s, t, a \} \) and the operations \(*\) and \# defined by the accompanying tables.

\[
\begin{array}{c|cccc}
* & r & s & t & a \\
\hline
r & a & r & s & t \\
s & r & s & t & a \\
t & s & t & a & r \\
a & t & a & r & s
\end{array}
\quad
\begin{array}{c|cccc}
\# & r & s & t & a \\
\hline
r & t & a & r & s \\
s & a & r & s & t \\
t & r & s & t & a \\
a & s & t & a & r
\end{array}
\]

\(a\) What is the identity element for *? [2]

\(b\) What is the inverse of \(a\) under the operation #? [2]

\(c\) Find the value of \((r \ast t) \# (a \ast s)\). [3]

\(d\) Solve for \(x\): \((t \ast s) \# x = r\). [3]

37 In a box, there are five balls. Three are red and two are white.

\(a\) Two balls are drawn without replacement. Find the probability that

(1) both are red [2]
(2) both are white [2]
(3) one is red and one is white [3]

\(b\) Three balls are drawn without replacement. Find the probability of drawing a white ball followed by two red balls. [3]

38 In right triangle \(\triangle ABC\), altitude \(\overline{CD}\) is drawn to hypotenuse \(\overline{AB}\). If \(AB\) is 4 times as large as \(AD\) and \(AC\) is 3 more than \(AD\), find the length of \(\overline{AD}\).

[Only an algebraic solution will be accepted.] [4, 6]

39 Given: Bill vacations in Canada or the United States.
   If the metric system is not used, then gasoline is not sold in liters.
   If Bill vacations in Canada, then gasoline is sold in liters.
   Bill does not vacation in the United States.

Let \(C\) represent: “Bill vacations in Canada.”
Let \(S\) represent: “Bill vacations in the United States.”
Let \(M\) represent: “The metric system is used.”
Let \(L\) represent: “Gasoline is sold in liters.”

Prove: The metric system is used. [10]

40 \(a\) Draw a graph of the equation \(y = x^2 - 4x + 3\) for all values of \(x\) such that \(-1 \leq x \leq 5\). [6]

\(b\) On the same set of axes, sketch the graph of the equation \(x^2 + (y - 3)^2 = 4\). [3]

\(c\) Using the graphs drawn in parts \(a\) and \(b\), determine the number of points these graphs have in common. [1]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed. [10]

41 Quadrilateral $KLMN$ has coordinates $K(-2,3), L(4,6), M(3,2),$ and $N(-3,-1)$. Using coordinate geometry, prove that
   
   a) the diagonals bisect each other [4]
   b) the opposite sides are congruent [6]

42 Given: quadrilateral $ABCD$, $AEC$, $BED$, $AB \cong AD$, and $BC \cong DC$.

Prove: $\angle 1 \cong \angle 2$ [10]
The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION  
SEQUENTIAL MATH — COURSE II  
Tuesday, June 21, 1988—1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil ........................................................... Teacher. ...........................................................

School ........................................................... Grade. ...........................................................

Your answers to Part I should be recorded on this answer sheet.

Part I  
Answer 30 questions from this part.

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<th>Answer</th>
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35 Answer question 35 on the other side of this sheet.
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Math.-Course II-June '88
FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Tuesday, June 21, 1988 — 1:15 to 4:15 p.m., only

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 15-34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 21  (11) 6  (21) 2  (31) 3
(2) 135  (12) 4/5  (22) 3  (32) 1
(3) 120  (13) 4  (23) 4  (33) 1
(4) 0  (14) 1  (24) 2  (34) 3
(5) 10  (15) 1  (25) 2  (35) construction
(6) 8  (16) 3  (26) 4
(7) 6  (17) 3  (27) 3
(8) 3  (18) 4  (28) 4
(9) 12  (19) 1  (29) 1
(10) 3\sqrt{2}  (20) 2  (30) 4

[OVER]
Please refer to the Department publication *Guide for Rating Regents Examinations in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) $a$ $s$ [2] 
    $b$ $s$ [2] 
    $c$ $t$ [3] 
    $d$ $r$ [3] 

(37) $a$ (1) $\frac{3}{10}$ [2] 
    (2) $\frac{1}{10}$ [2] 
    (3) $\frac{2}{5}$ [3] 
    $b$ $\frac{1}{5}$ [3] 

(38) 3 [4,6] 

(40) $c$ 2 [1]