New York State Education Department

Three-year Sequence for High School Mathematics

Course II

Final Examination

In Lieu of the Tenth Year Mathematics Regents Examination

Thursday, January 25, 1979—1:15 to 4:15 p.m., only

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

Part I

Answer only 30 of the 35 questions in this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 The measure of angle A is 15 degrees more than twice its supplement. Find m\(\angle A\).

2 What are the coordinates of the midpoint of the line segment joining the points whose coordinates are (-2,3) and (4,-3)?

3 What is the slope of the line that passes through the points (4,9) and (-1,12)?

4 In the accompanying figure, \(\overline{AC} \parallel \overline{BD}\), \(AC = 3\), \(BD = 4\), and \(AE = 12\). Find \(BE\).

5 The measures of two opposite angles of a parallelogram are represented by \(3x + 40\) and \(x + 50\). Find \(x\).
6 In the accompanying figure, \( \overline{AB} \parallel \overline{CD} \)
and \( \overline{EF} \) intersects \( \overline{AB} \) at \( E \) and \( \overline{CD} \) at \( F \).
If \( m_\angle AEF \) is represented by \( 3x \) and
\( m_\angle EFD = 60 \), find the value of \( x \).

7 Write an equation of the circle whose center is at \((0,2)\) and whose
radius is of length 4.

8 The altitude to the hypotenuse of a right triangle divides the
hypotenuse into two segments. If the length of the altitude is
6 and the length of the longer segment is 9, what is the length
of the shorter segment?

9 What is the length of the radius of a circle whose center is the
origin and which passes through the point \((3,4)\)?

10 Write an equation of the locus of points that are equidistant from
the points \((0,6)\) and \((0, -2)\).

11 The lengths of the bases of a trapezoid are 4 and 8. If the length
of the altitude of the trapezoid is 3, find the area of the trape-
zoïd.

12 How many different 6-letter permutations are there from the word
"FOLLOW"?

13 An urn contains 4 green marbles and 2 white marbles, all of equal
size. If two marbles are selected at random with no replacement,
what is the probability that both marbles selected are the same
color?

14 Find the positive root of \( 2x^2 + 3x - 5 = 0 \).

15 Determine the value of \( e \times (c \times e) \) within the following system:

\[
\begin{array}{c|cccc}
* & e & a & b & c \\
e & e & a & b & c \\
a & a & b & c & e \\
b & b & c & e & a \\
c & c & e & a & b \\
\end{array}
\]
16 Given the set \{w, x, y, z\} and the operation \# as shown in the table. What is the identity element for the set?

<table>
<thead>
<tr>
<th>#</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
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<tbody>
<tr>
<td>w</td>
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<td>y</td>
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<tr>
<td>z</td>
<td>y</td>
<td>w</td>
<td>z</td>
<td>x</td>
</tr>
</tbody>
</table>

Directions (17-34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

17 If the measures of three angles of a triangle are represented by \(x + 30\), \(4x + 30\), and \(10x - 30\), the triangle must be

(1) obtuse
(2) isosceles
(3) right
(4) scalene

18 The corresponding altitudes of two similar triangles are 6 and 14. If the perimeter of the first triangle is 21, what is the perimeter of the second triangle?

(1) 9
(2) 27
(3) 49
(4) 64

19 The lengths of the sides of a right triangle may be

(1) 5, 7, 8
(2) 7, 8, 12
(3) 7, 9, 11
(4) 8, 15, 17

20 What is the total number of points in a plane 5 units from a given line and also 5 units from a given point on the line?

(1) 1
(2) 2
(3) 3
(4) 4

21 What is the length of the altitude of an equilateral triangle whose side is of length 4?

(1) \(2\sqrt{3}\)
(2) 2
(3) \(4\sqrt{3}\)
(4) 4
22 In the accompanying figure of triangle ABC, BD is drawn so that \( \overline{AB} \cong \overline{BD} \). If \( m\angle ABD = 80 \), then \( m\angle BDC \) is

\[
\begin{array}{ll}
(1) & 50 \\
(2) & 100 \\
(3) & 130 \\
(4) & 150
\end{array}
\]

23 The negation of \( p \lor \neg q \) is

\[
\begin{array}{ll}
(1) & \neg p \lor q \\
(2) & \neg p \land q \\
(3) & p \land \neg q \\
(4) & \neg p \lor \neg q
\end{array}
\]

24 Which is the negation of the statement, "All squares are rectangles"?

\[
\begin{array}{ll}
(1) & \text{All rectangles are squares.} \\
(2) & \text{No squares are rectangles.} \\
(3) & \text{Some squares are rectangles.} \\
(4) & \text{Some squares are not rectangles.}
\end{array}
\]

25 Which is logically equivalent to the statement, "If I eat, then I live"?

\[
\begin{array}{ll}
(1) & \text{If I do not eat, then I do not live.} \\
(2) & \text{If I do not live, then I do not eat.} \\
(3) & \text{If I live, then I eat.} \\
(4) & \text{If I do not eat, then I live.}
\end{array}
\]

26 Under which operation is the set of positive rational numbers not closed?

\[
\begin{array}{ll}
(1) & \text{addition} \\
(2) & \text{subtraction} \\
(3) & \text{multiplication} \\
(4) & \text{division}
\end{array}
\]

27 Which is an equation of the axis of symmetry of the graph \( y = x^2 + 8x - 10 \)?

\[
\begin{array}{ll}
(1) & y = -4 \\
(2) & y = 4 \\
(3) & x = -4 \\
(4) & x = 4
\end{array}
\]
28 Which is an equation for the line that is parallel to $2x - y = 8$ and passes through the point (0,2)?

(1) $y = x + 4$
(2) $y = 2x + 2$
(3) $y = -2x + 2$
(4) $y = -x + 2$

29 If a card is drawn from a deck of 52 playing cards, the probability that it is an ace or a club is

(1) $\frac{1}{52}$
(2) $\frac{16}{52}$
(3) $\frac{17}{52}$
(4) $\frac{48}{52}$

30 Which property is not necessary in order for a system to be a group?

(1) existence of an inverse
(2) closure
(3) associativity
(4) commutativity

31 For the equation $x^2 - 6x - 2 = 0$, $x$ is equal to

(1) $\frac{6 \pm \sqrt{28}}{2}$
(2) $\frac{-6 \pm \sqrt{28}}{2}$
(3) $\frac{6 \pm \sqrt{44}}{2}$
(4) $\frac{-6 \pm \sqrt{44}}{2}$

32 In the accompanying diagram, ABCD is a rhombus with diagonal DD. Which is not true?

(1) $\overline{AB} \cong \overline{BC}$
(2) $\overline{AD} \cong \overline{DB}$
(3) $m\angle ABD = m\angle DBC$
(4) $m\angle ABD = m\angle ADB$

33 If $a + b$ and $-c + -b$, then it logically follows that

(1) $a + c$
(2) $b + a$
(3) $c + a$
(4) $c + b$
34 Which is an equation of the graph shown at the right?

1. \( y = x^2 - 2x + 1 \)
2. \( y = -x^2 - 2x + 1 \)
3. \( y = x^2 + 2x + 1 \)
4. \( y = -x^2 + 2x + 1 \)

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct an altitude of trapezoid ABCD.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

36 Given the equation: \( y = x^2 - 4x + 3 \)

a. Draw the graph of the equation in the interval \( 0 \leq x \leq 4 \). [6]

b. Write an equation of the axis of symmetry. [2]

c. Using the graph from part a, find the roots of \( x^2 - 4x + 3 = 0 \).

37 Triangle ABC has vertices A(-6,-4), B(4,2), and C(0,4).

a. On graph paper, draw triangle ABC. [2]

b. Find the area of triangle ABC. [8]
38 Base your answers to a through e on the table below which represents addition modulo 4.

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<td>3</td>
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<td>1</td>
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<tr>
<td>3</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

a What is the identity element? [2]
b What is the value of 3 + 2? [2]
c What is the value of 3 + 3 + 3? [2]
d What is the inverse of 2? [2]
e Solve for x: 3 + x = 1 + 1 [2]

39 From a standard deck of 52 cards, two cards are drawn at random with no replacement. State the probability of selecting:

a the ace of spades and king of spades (in that order) [2]
b a pair of aces [2]
c any ace and any king (in that order) [2]
d any ace and any king (in either order) [2]
e two spades [2]

40 In right triangle ABC, angle C is a right angle. If AC is 1 more than BC and AB is 2 more than BC, find the lengths of the three sides of triangle ABC. [Only an algebraic solution will be accepted.] [5,5]

41 On your answer paper, write the letters a through e. After each letter, state whether the conclusion is valid or invalid.

a If you have the radio turned up all the way, then you cannot hear the phone ring.  
   You do not have the radio turned up all the way.  
   Conclusion: You can hear the phone ring. [2]

b The winner of the Dingbat award gets a year’s supply of pretzels.  
   I won a year’s supply of pretzels.  
   Conclusion: I won the Dingbat award. [2]

c Logic exercises are for the birds.  
   What is for the birds is not for me.  
   Conclusion: Logic exercises are not for me. [2]

d $p \lor q$  
   $\neg q$  
   Conclusion: $p$ [2]

e If a plant blooms, it is well nourished.  
   If a plant is not watered, it is not well nourished.  
   Conclusion: If a plant blooms, it has been watered. [2]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed.

42 The coordinates of the vertices of triangle ABC are A(6,2), B(-4,4), and C(-2,-4).
   a Find the coordinates of the midpoint D of side AB.  [2]
   b Find the coordinates of the midpoint E of side BC.  [2]
   c By means of coordinate geometry, show that DE || AC and state a reason for your conclusion.  [6]

43 Given the following sentences:
   Alpha is first or Beta is second.
   If Beta is second, then Gamma is third.
   If Gamma is third, then Delta is home.
   Delta is not home.
   Therefore, Alpha is first.

   Let A represent:  "Alpha is first."
   Let B represent:  "Beta is second."
   Let G represent:  "Gamma is third."
   Let D represent:  "Delta is home."

   a Using A, B, G, and D and proper logic connectives, express each sentence in symbolic form.  [5]
   b Using laws of inference, show that a valid conclusion has been reached.  [5]

44 Given:  FH = HG, DH = HE, \angle 1 = \angle 2

Prove:  \angle A = \angle C  [10]
New York State Education Department
Three-year Sequence for High School Mathematics
Course II
Final Examination

Thursday, January 25, 1979—1:15 to 4:15 p.m., only

Pupil........................................Teacher..........................
School..........................................................

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer only 30 questions in this part.

1 13 25
2 14 26
3 15 27
4 16 28
5 17 29
6 18 30
7 19 31
8 20 32
9 21 33
10 22 34
11 23 35 Answer question 35 on the other side of this sheet.
12 24

[OVER]
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature
For Teachers Only
Scoring Key
Three-year Sequence for High School Mathematics

Course II
Thursday, January 25, 1979 - 1:15 to 4:15 p.m., only

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] For questions 17-34, allow credit if the pupil has written the correct answer instead of the numeral 1,2,3 or 4.

(1) 125
(2) (1,0) or \( \frac{x}{y} = 1 \)
(3) \(-\frac{3}{5}\)
(4) 16
(5) 5
(6) 20
(7) \(x^2 + (y - 2)^2 = 16\)
(8) 4
(9) 5
(10) \(y = 2\)
(11) 18
(12) 180
(13) \(\frac{7}{15}\) or \(\frac{14}{30}\)
(14) 1
(15) c
(16) y
(17) 2
(18) 3
(19) 4
(20) 2
(21) 1
(22) 3
(23) 2
(24) 4
(25) 2
(26) 2
(27) 3
(28) 2
(29) 2
(30) 4
(31) 3
(32) 2
(33) 1
(34) 3
(35) construction

[OVER!]

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General Directions - Parts II and III

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(36) b \( x = 2 \) [2]
    c \( x = 1 \) [2]  
    x = 3 [2]

(37) b 22 [8]

(38) a 0 [2]
    b 1 [2]
    c 1 [2]
    d 2 [2]
    e 3 [2]

(39) a \( \frac{1}{2,652} \) or \( \frac{1}{52} \cdot \frac{1}{51} \) [2]
    b \( \frac{12}{2,652} \) or \( \frac{4}{52} \cdot \frac{3}{51} \) [2]
    c \( \frac{16}{2,652} \) or \( \frac{4}{52} \cdot \frac{4}{51} \) [2]
    d \( \frac{32}{2,652} \) or \( 2 \cdot \frac{4}{52} \cdot \frac{4}{51} \) [2]
    e \( \frac{156}{2,652} \) or \( \frac{13}{52} \cdot \frac{12}{51} \) [2]

(40) Analysis [5]
    BC = 3
    AC = 4 [5]
    AB = 5

(41) a invalid [2]
    b invalid [2]
    c valid [2]
    d valid [2]
    e valid [2]

Part III

(42) a \((1,3)\) or \(x = 1, y = 3\) [2]

b \((-3,0)\) or \(x = -3, y = 0\) [2]

(43) a A \lor B [1]
    b A \lor B [1]
    c A \lor B [1]
    \(\neg B\) [1]
    A [1]