The University of the State of New York

264th High School Examination

PLANE TRIGONOMETRY

Tuesday, August 20, 1935 — 3.30 to 6.30 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane trigonometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1935.

The minimum time requirement previous to entering summer high school is five recitations a week for half a school year, or the equivalent.

For those pupils who have met the time requirement previous to entering summer high school the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1935 is required.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Plane Trigonometry

See instructions for groups II and III on page 1.

Group II

Answer two questions from this group.

21  a Using triangle $ABC$, in which $A$ is obtuse, derive the formula
     \[ a^2 = b^2 + c^2 - 2bc \cos A \]  \[10\]
     b Find $A$ correct to the nearest degree, when $a = 4$, $b = 2$, $c = 3$  \[2\frac{1}{2}\]

22  a Solve for all values of $x$ between $0^\circ$ and $360^\circ$:
     \[ \cos 2x + \cos x = 0 \]  \[6\frac{1}{2}\]
     b Prove the identity:
     \[ \sin \left( \frac{\pi}{4} + x \right) - \sin \left( \frac{\pi}{4} - x \right) = \sqrt{2} \sin x \]  \[6\]

23  a Draw the graph of \( y = 2 \sin 2x \) from \( x = 0^\circ \) to \( x = 90^\circ \) at intervals of 15°.  \[10\frac{1}{2}\]
     b From the graph drawn in answer to a, find the value of \( 2 \sin 2x \), when \( x = 85^\circ \)  \[2\]

*24  a Represent graphically the complex number \( -1 - i \)  \[2\]
     b Transform \( -1 - i \) into the form \( r (\cos \theta + i \sin \theta) \)  \[8\frac{1}{2}\]
     c What is the modulus of \( -1 - i \)?  \[1\]
     d What is the amplitude of \( -1 - i \)?  \[1\]

Group III

Answer two questions from this group.

25  From two successive milestones $A$ and $B$ on a straight highway, a barn $C$ is sighted. Angle $CAB$ is $60^\circ$ and angle $CBA$ is $55^\circ$. Find the length of the shortest road that can be built from the barn to the highway.  \[12\frac{1}{2}\]

26  A monument stands on a horizontal plane and may be approached by a straight, narrow path. At a point $A$ on this path the angle of elevation of the top of the monument is $m^\circ$ and at a point $B$ on the same path and $c$ feet nearer the monument the angle of elevation is $n^\circ$. Derive a formula for the height of the monument in terms of $c$, $m$ and $n$.  \[12\frac{1}{2}\]

27  In order to find the distance $AB$ across a lake, a convenient point $C$ was chosen and the following measurements were made: $CA = 348.5$ yards, $CB = 268.5$ yards, angle $ACB = 61^\circ 40'$; what is the distance from $A$ to $B$?  \[12\frac{1}{2}\]

* This question is based on one of the optional topics in the syllabus.
Name of school..........................................................Name of pupil..................................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2\(\frac{1}{2}\) credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find \(\log \sin 48^\circ 52'\)

2 \(\log \cos A = 9.8436 - 10\); find, correct to the nearest minute, the smallest positive value of \(A\).

3 Find \(\log \cot 55^\circ 26'\)

4 \(\log \tan B = 9.7589 - 10\); find, correct to the nearest minute, the value of \(B\) if \(B\) is an angle in the first quadrant.

5 Express \(\sec x\) in terms of \(\sin x\), where \(x\) is an angle in the first quadrant.

6 Express \(\sin 192^\circ\) as a function of a positive acute angle.

7 As a positive angle increases in the fourth quadrant, does its tangent increase or decrease?

8 Express \(\frac{5\pi}{4}\) radians in degrees.

9 Find, correct to the nearest degree, one value of \(\sin^{-1} \frac{1}{2}\).

10 \(\cos A = \frac{1}{2}\); find \(\tan A\) when \(A\) is an acute angle.

11 Find the value of \(\cos (-38^\circ 10')\)

12 Find the exact value of \(\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ\)

13 What is the resultant of forces of 12 pounds and 5 pounds acting at right angles to each other?

14 Find the smallest positive angle that satisfies the equation \(\sec x = \csc x\)

15 Express \(\cot 2x\) in terms of \(\tan x\).

16 Find the radius of the circle in which a central angle of 3 radians intercepts an arc 1\(\frac{1}{2}\) inches long.

Directions (questions 17-20) — Indicate whether each of the following statements is always true, sometimes true or never true by writing the word always, sometimes or never on the dotted line at the right.

17 As a positive angle increases, its cosine decreases.

\[\frac{\tan x}{\sin x} = \sec x\]

19 \(\sin 3x = 3\)

20 In triangle \(ABC\), \(\sin (A + B) = \sin C\)