The University of the State of New York

278TH HIGH SCHOOL EXAMINATION

PLANE TRIGONOMETRY

Thursday, June 20, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

Fill in the following lines:

Name of school.................................... Name of pupil....................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive **$2\frac{1}{2}$** *credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.*

1 Find the numerical value of $\sin \frac{\pi}{2}$. 1................

2 Find the numerical value of $\tan(-135°)$. 2................

3 Find, correct to the *nearest minute,* the positive acute angle A when $A = \cos^{-1} .9381$ 3................

4 Find log sin 34° 16′ 4................

5 Find, correct to the *nearest hundredth,* the number whose logarithm is 1.7060. 5................

6 Express the cotangent of a positive acute angle A in terms of the sine of A. 6................

7 Write the value of the cosine of an acute angle whose tangent is $\frac{12}{5}$. 7................

8 What is the minimum positive value of $3 \sec 2x$? 8................

9 If $\sin x = a$, express $\cos 2x$ in terms of a. 9................

10 If $\cos A = b$, express $\sin^2 \frac{1}{2} A$ in terms of b. 10................

11 If $\log a = 4.2484$ and $\log b = 3.1242$, find $\log \frac{b^2}{a}$. 11................

12 In $\triangle ABC$, express c in terms of a, $\sin A$ and $\sin C$. 12................

13 In $\triangle ABC$, $a = 6$, $b = 3$, $c = 8$; find $\cos A$. [Answer may be left in fractional form.] 13................

14 In $\triangle ABC$, $C = 60°$ and $\frac{a+b}{a-b} = \frac{\sqrt{3}}{1}$; find $\tan \frac{1}{2}(A - B)$. 14................

15 Find the value of x between 0° and 90° which satisfies the equation $\tan^2 x - \tan x = 0$. 15................

16 In $\triangle ABC$, $C = 90°$, $c = 10$, $A = 22° 30'$; find b correct to the *nearest integer.* 16................

Directions (questions 17–20) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

17 The statement $\tan A \sin 2A = 2 \sin^2 A$ is true for (a) only one value of A, (b) no value of A or (c) all values of A. 17................

18 Given $A = 30°$, $c = 10$, $a = 12$; then (a) only one triangle can be constructed with the given parts, (b) two such triangles are possible or (c) no such triangle exists. 18................

19 As $\cos A$ increases from -1 to 0, $\csc A$ (a) decreases from ∞ to 1, (b) increases from $-\infty$ to -1 or (c) decreases from -1 to $-\infty$. 19................

20 As x varies from 0° to 270°, the graphs of the functions $y = \tan x$ and $y = \cos x$ intersect in (a) one point, (b) two points or (c) three points. 20................

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 *a* Prove the identity: $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$ [5]

b Find the positive angle less than 180° which satisfies the equation $3 \cos^2 x + 2 \sin x - 2 = 0$ [5]

22 Derive the formula for the area of triangle ABC in terms of b, c and A. [Consider only the case where A is obtuse.] [10]

23 *a* Derive the relationship $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$ for the case where A and C of $\triangle ABC$ are acute. [4]

b Starting with the formulas for $\sin (x + y)$ and $\cos (x + y)$, derive the formula for $\tan (x + y)$. [6]

24 *a* Draw and letter clearly the line values of the six trigonometric functions of an angle in the second quadrant. [4]

b For *each* function indicate the line segment representing it and state whether the line segment is positive or negative. [6]

*25 Using De Moivre's Theorem, express $(2 + 2i\sqrt{3})^4$ in the form $a + bi$. [10]

Group III

Answer at least two questions from this group.

26 In $\triangle ABC$, $c = 28.7$, $a = 36.3$, $A = 50° \ 25'$; find C correct to the *nearest minute*. [10]

27 In $\triangle ABC$, $a = 10.26$, $b = 15.50$, $c = 18.24$; also

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)\ (s-c)}{s\ (s-a)}}, \text{ where } s = \tfrac{1}{2}\ (a + b + c) = 22$$

Fill in the following outline, finding A correct to the *nearest minute*: [10]

$\log (s - b) =$ ....

$\log (s - c) =$ ....

$\log (s - b)\ (s - c) =$ ....

$\log s =$ ....

$\log (s - a) =$ ....

$\log s\ (s - a) =$ ....

$\log \dfrac{(s-b)\ (s-c)}{s\ (s-a)} =$ ....

$\log \tan \dfrac{A}{2} =$ ....

$\dfrac{A}{2} =$ ....

$A =$ ....

28 Two observers 5280 feet apart on a straight horizontal road observe a balloon between them directly above the road. At the points of observation the angles of elevation of the balloon are 60° and 75°. Find, correct to the *nearest* foot, the height of the balloon. [10]

29 A and B are points on opposite sides of a lake at its greatest width. A point C is 2820 feet from B and 2240 feet from A; the angle ACB is 64°. Find, correct to the *nearest foot*, the greatest width of the lake. [10]

* This question is based on one of the optional topics in the syllabus.