Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one hour. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one-hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.
Part I

Answer all questions in this part. Each question has 3 credits assigned to it. Each answer must be reduced to its simplest form.

1 Complete the following statement: The maximum value for sine $A$ as $A$ varies from $0^\circ$ to $360^\circ$ is ....

2 In a right triangle, $\csc A = 1\frac{3}{5}$, $c = 50$; find $b$.

3 In a right triangle, $b = 15$, $c = 15\sqrt{2}$; find angle $A$.

4 Two campers wish to know approximately how wide the river is at a signal tower diagonally across from their camp. They find the angle made by a straight line to the base of the tower with a straight line along the bank to be $60^\circ$. They go along this line on the bank until they find a point from which another line to the tower also makes an angle of $60^\circ$ with the line along the bank. The distance between the two points on the bank is 200 feet. Find to the nearest foot the width of the river at the tower.

5 A ladder 24 feet long, standing on level ground, rests against the side of a building at a point 20 feet from the ground. What angle does the ladder make with the side of the building? [By means of natural functions find the answer to the nearest minute.]

6 In a right triangle, $\sin A = \frac{3}{5}$; find $\tan A$.

7 Express $\cos 215^\circ$ in terms of a positive angle less than $90^\circ$.

8 Solve for the positive acute value of $x$:
\[2 \sin^2 x + 5 \sin x = 3\]

9 Given $\cos x = \frac{1}{2}$, $x$ being an angle in the first quadrant; find $\tan 2x$.

10 Write the formula used in changing radians to degrees.
Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.
The minimum time requirement for plane trigonometry is five recitations a week for half a school year, or the equivalent.

Part II
Answer five questions from this part, selecting three questions from group I and two from group II.

Group I
Answer three questions from this group.

11 To eliminate a grade crossing, a bridge is built over some railroad tracks. The approach to the bridge on one side starts at a point $A$ on a level with the tracks and extends to a point $B$ at which the bridge begins. $B$ is 32 feet above a point $C$ on a level with the tracks. The inclination of the approach is $6^\circ 30' 16''$.
   a) Find the length of $AC$. [7]
   b) Find the length of the approach. [7]

12 A surveyor wishes to plot a piece of ground of irregular shape. He starts at a point $A$ and runs a line due west 40 rods to a stake $B$, thence 30 rods due north to $C$, thence 60 rods in a direction somewhat east from north to $D$, thence 75 rods to the starting point $A$. Find angle $D$ correct to the nearest minute. [14]

13 Two sides of a triangular plot of ground are 160.6 feet and 186.4 feet and the angle included between them is $40^\circ 28'$.
   a) Write a suitable formula for finding the area of the plot. [3]
   b) Find the number of square yards in the plot. [11]

14 Two straight railroad tracks that form with each other an angle of $70^\circ$, start from the same station. Two trains leave this station at the same time, one on each track, and run at the rates of 30 and 48 miles an hour. What will be the distance between the trains at the end of 20 minutes? [14]

Group II
Answer two questions from this group.

15 Derive the formulas for $\tan (x - y)$ and $\cot (x - y)$, starting with the formulas for the sine and cosine of the difference between two angles. [7, 7]

16 Copy the following statements and fill the blanks with the correct words or symbols:
   a) If in a unit circle the trigonometric functions are indicated by lines, it is readily seen that the sign of both tangent and cotangent is . . . in the second quadrant and . . . in the third quadrant. [1, 1]
   b) The statement that the . . . of a triangle are proportional to the . . . of the opposite angles is called the law of . . . [3]
   c) The law of tangents is that the . . . of two sides of a triangle is to their . . . as the . . . of half the sum of the opposite . . . is to the . . . of half their difference. [3]
   d) The number $-\frac{1}{2}\sqrt{3}$ is the cosine of an obtuse angle of . . . degrees. [3]
   e) The . . . of $15^\circ = \sqrt{\frac{1-\cos 30^\circ}{2}}$ [3]

17 Prove the following identities:
   \[
   (1 - \sin^2 A) \tan^2 A = \sin^2 A \quad [7]
   \]
   \[
   \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} \quad [7]
   \]

18 a) Complete the following table, determining from the table of natural functions the values of tan $x$ to the nearest tenth: [7]
   \[
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   x & -60^\circ & -45^\circ & -30^\circ & 0^\circ & 30^\circ & 45^\circ & 60^\circ \\
   \hline
   \hline
   \end{array}
   \]
   b) From the table plot the graph of $y = \tan x$ from $x = -60^\circ$ to $x = 60^\circ$. [6]
   c) Does $\tan x$ increase or decrease algebraically as $x$ increases from $-60^\circ$ to $0^\circ$? [1] [3]