Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and four questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. $A$, $B$ and $C$ represent the angles of a triangle $ABC$; $a$, $b$ and $c$ represent the respective opposite sides. In a right triangle, $C$ represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school..............................................Name of pupil..............................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it. Each answer must be reduced to its simplest form.

1. Express 150° in radian measure. [Answer may be left in terms of \( \pi \).]

   Ans.................................

2. Find \( \log \sin 42°.24' \)

   Ans.................................

3. Given \( \log \tan A = 9.5857 - 10 \); find, correct to the nearest minute, the value of \( A \) if \( A \) is a positive acute angle.

   Ans.................................

4. Find the value of \( \sin 210° \).

   Ans.................................

5. In triangle \( ABC \), \( C = 90° \), \( \sin A = \frac{1}{8} \), \( a = 4 \); find \( c \).

   Ans.................................

6. Express \( \cos 2A \) in terms of \( \cos A \).

   Ans.................................

7. Given \( \tan A = r \) and \( \tan B = s \); express \( \tan (A - B) \) as a function of \( r \) and \( s \).

   Ans.................................

8. Express \( \tan A \) in terms of \( \sin A \) where \( A \) is an angle in the first quadrant.

   Ans.................................

9. Given \( \cos y = -\frac{5}{8} \) and \( y \) an angle in the second quadrant; find the value of \( \sin 2y \).

   Ans.................................

10. In triangle \( ABC \), \( \frac{1}{2} (A + B) = 60° \) and \( \frac{1}{2} (A - B) = 45° \); find the value of the ratio \( \frac{a + b}{a - b} \)

   Ans.................................

11. A road rises 24 feet in a horizontal distance of 300 feet. Find, correct to the nearest degree, the angle that the road makes with the horizontal.

   Ans.................................

12. Find, correct to the nearest minute, the value of \( A \) in the first quadrant that satisfies the equation \( \tan^2 A = 4 \)

   Ans.................................

13. Find the value of \( \cos (-63°) \)

   Ans.................................

14. Given two acute angles \( x \) and \( y \) such that \( \sin x = \frac{3}{8} \) and \( \cos y = \frac{1}{8} \); find the value of \( \sin (x - y) \).

   Ans.................................

15. Find the positive value of \( \cos (\sin^{-1} \frac{1}{3}) \)

   Ans.................................

16. The sides of a triangle are 5, 7 and 9; find the value of the cosine of the largest angle.

   Ans.................................

17. How many solutions are there for triangle \( ABC \), if \( A = 43° \), \( a = 15 \), \( b = 10 \)?

   Ans.................................

18. In which quadrant do both the sine and the cosine of a positive angle increase as the angle increases?

   Ans.................................

19. Which of the statements below is never true?

   a. \( \sin x = \cos x \)
   b. \( \sin \frac{1}{2}x = \frac{1}{2} \)
   c. \( \tan x = -0.3245 \)
   d. \( \cos 2x = 2 \)

   Ans.................................

20. What is the value of \( \sin^2 3x + \cos^2 3x \)?

   Ans.................................
Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in plane trigonometry.

The minimum time requirement for plane trigonometry is five recitations a week for half a school year, or the equivalent.

Part II

Answer four questions from this part, selecting two questions from each group.

Group I

Answer two questions from this group.

21 a Assuming the formulas for \( \sin (x + y) \) and \( \cos (x + y) \), derive the formula for \( \tan (x + y) \). \([8]\)

b Using the formula for \( \tan (x + y) \), derive the formula for \( \tan 2x \). \([4]\)

22 a Using triangle \( ABC \) in which \( A \) is obtuse, derive the formula

\[
\frac{\sin A}{\sin B} = \frac{a}{b} \quad \text{[7]}
\]

b Prove the identity:

\[
\sec^2 \frac{1}{2}x = \frac{2(1 - \cos x)}{\sin^2 x} \quad \text{[5]}
\]

23 a Find, correct to the nearest minute, the value of \( x \) between 0° and 30° that satisfies the equation \( 6 \cos^2 x + 5 \sin x - 7 = 0 \). \([8\frac{1}{2}]\)

b Prove the identity: \( \cot 2A = \frac{(\cot A - \tan A)}{2} \). \([4]\)

Group II

Answer two questions from this group.

24 In triangle \( ABC \), \( a = 6.73 \), \( b = 4.52 \), \( c = 3.75 \); find \( B \) correct to the nearest minute. \([12\frac{1}{2}]\)

25 Two observers along a straight coast line, stationed 3.2 miles apart, detect a burning ship \( S \). The first observer, who is at \( A \), finds that the angle formed by the line of sight \( AS \) with the shore line \( AB \) is 74° 37'; the second observer, at \( B \), finds that the angle formed by the line of sight \( BS \) and the shore line \( BA \) is 82° 52'. How far is the ship from the nearest point on shore? \([12\frac{1}{2}]\)

26 Three towns, \( A \), \( B \) and \( C \), are the vertices of a triangle \( ABC \), in which \( AC = 6.4 \) miles, \( BC = 3.6 \) miles and angle \( B = 85° 10' \). Two men leave \( A \) at the same time to go to \( C \). The first travels along \( AC \) at the rate of 30 miles an hour; the second travels along \( ABC \) at the rate of 60 miles an hour. Which arrives at \( C \) first and by how many minutes? \([12\frac{1}{2}]\)