

## High School Department

155TH EXAMINATION

## PLANE GEOMETRY

August 1898—Three hours, only

Answer eight questions including one from each of the three divisions. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive  $12\frac{1}{2}$  credits. Papers entitled to 75 or more credits will be accepted.

First division 1 Define *plane angle, polygon, axiom, secant, apothem*. Illustrate each.

2 Prove that the sum of the interior angles of a triangle is equal to two right angles and state a theorem in the demonstration of which this fact is used.

3 Prove that parallel lines intercept equal arcs of a circumference.

4 Prove that in any triangle the square on the side opposite an acute angle is equal to the sum of the squares on the other two sides diminished by twice the product of one of those sides by the projection of the other on that side.

5 Assuming that the area of a regular polygon equals one half the perimeter by the apothem, and the circumference of a circle equals  $2\pi r$ , prove that the area of the circle is equal to  $\pi r^2$ . What is  $\pi$ ?

Second division 6 The difference of two adjacent interior angles of a parallelogram is  $A$ ; find the value of each angle in terms of  $A$ .

7 Find the length of the longest and of the shortest chord that can be drawn through a point 5 inches from the center of a circle whose radius is 13.

8 In a circle whose radius is 7 find the area of the segment whose arc is  $90^\circ$ .

9 Given a right triangle whose hypotenuse is 10 and shortest side 6; find the perimeter and area of a similar triangle whose shortest side is 2.

10 Find the side of a square equivalent to a trapezoid whose upper base is 10, altitude 12 and legs each 13.

Third division 11 Construct a line which shall be to a given line as  $\sqrt{3}:1$ .

12 If one leg of an isosceles triangle is produced its own length through the vertex, prove that the line drawn from its extremity to meet the other leg at the base is perpendicular to the base.

13 Construct a circle with a given radius tangent to two intersecting lines.

14 Prove that the sum of two opposite angles of an inscribed quadrilateral equals the sum of the other two.

15 Construct an equilateral triangle equivalent to a given square.