The University of the State of New York

310TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Wednesday, August 23, 1950 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1950 or number and length in minutes of lessons taken in the summer of 1950 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1950 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Prove that the sum of the angles of a triangle is a straight angle. [10]

27 Quadrilateral $ABCD$ is inscribed in a circle. Side AD is equal to side BC. Straight lines are drawn from E, the mid-point of side AB, to D and C.

a Prove arc $ADC =$ arc BCD [3]

b Prove $DE = CE$ [7]

28 In triangle ABC, angle C is a right angle, and D is a point on AC. With AD as the diameter, a circle is drawn cutting AB in E. Prove $AB \times AE = AC \times AD$. [10]

29 Prove that the area of a trapezoid is equal to one-half the product of its altitude and the sum of its bases. [10]

Part III

Answer one question from part III.

30 A circular fish pond is 38 feet in circumference. It is to be surrounded by a stone walk 4 feet wide.

a Find the radius of the pond to the *nearest foot.* [3]

b Using the value found in answer to *a,* find, to the *nearest dollar,* the cost of constructing the walk at 45 cents a square foot. [7]

31 In triangle ABC, AD is the altitude to base BC. $AB = 25$ feet, $AC = 26$ feet and $\angle B = 74°$

a Find AD to the *nearest foot.* [3]

b Find, to the *nearest square foot,* the area of triangle ABC. [7]

Part IV

Answer one question from part IV.

32 If the blank space in each of the following statements is filled by one of the words *always, sometimes,* or *never,* the resulting statement will be true. Write on your answer paper the letters *a, b, c, d, e* and opposite *each* write the word that will correctly complete the corresponding statement.

a The areas of two regular polygons ... have the same ratio as the squares of the corresponding sides. [2]

b If a parallelogram is inscribed in a circle, its diagonals are ... equal. [2]

c BD is a median of triangle ABC. If angle BDC is greater than angle BDA, BC is ... greater than AB. [2]

d The altitude to the hypotenuse of a right triangle is ... the mean proportional between the legs of the right triangle. [2]

e In triangle ABC, D is a point on side AB and E is a point on side BC. If $DE = \frac{1}{2} AC$, then DE is ... parallel to AC. [2]

33 Regular hexagon $ABCDEF$ is inscribed in a circle. Diagonals FB and FD intersect diagonal AE in G and H respectively.

a Prove that triangle FGH is an equilateral triangle. [3]

b Prove that $AG = GH = HE$ [5]

c If GH is 6 inches long, find the area of triangle AFE. [Answer may be left in radical form.] [2]

Plane Geometry

Fill in the following lines:

Name of pupil ....Name of school..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The vertex angle of an isosceles triangle is 80°. Find the number of degrees in an exterior angle formed by extending the base. 1................

2 The altitude upon the hypotenuse of a right triangle divides the hypotenuse into segments of 9 and 16. Find the length of the altitude. 2................

3 If two secants drawn to a circle from the same external point intercept arcs of 120° and 50° on the circle, find the number of degrees in the angle formed by the secants. 3................

4 Find the area of an equilateral triangle whose side is 8. [Answer may be left in radical form.] 4................

5 A square is circumscribed about a circle whose diameter is 8 inches. Find the area of the square. 5................

6 In parallelogram $ABCD$, angle B is 30° larger than angle A. Find the number of degrees in angle A. 6................

7 The diagonals of a rhombus are 18 and 24. Find a side of the rhombus. 7................

8 Secant ABC and tangent AD are drawn to a circle from an external point A. Chord $BC = 12$ inches and $AB = 4$ inches. Find the length of tangent AD. 8................

9 Two chords AB and CD of a circle intersect in E. If $AE = 4$, $EB = 6$ and $CE = 8$, find ED. 9................

10 In triangle ABC, DE which is parallel to BC cuts AB in D and AC in E. If $AD = 15$ inches, $DB = 12$ inches and $EC = 8$ inches, find the length of AE. 10................

11 Find the area of a trapezoid whose bases are 10 feet and 8 feet and whose altitude is 6 feet. 11................

12 If the sum of the interior angles of a polygon is 900°, find the number of sides of the polygon. 12................

13 Find the radius of a circle if the area of a 90° sector of that circle is 25π square inches. 13................

14 Find the length of an arc of 120° in a circle whose radius is 9 feet. [Answer may be left in terms of π.] 14................

15 In triangle ABC, angle $C = 90°$, $AB = 15$ inches and angle $A = 38°$. Find AC to the *nearest inch.* 15................

16 From a point P outside circle O, PO and tangent PT are drawn. If radius $OT = 5$ and $OP = 10$, how many degrees are there in angle OPT? 16................

17 The areas of two similar triangles are in the ratio 4:1. If a side of the larger triangle is 8 inches, find the length of the corresponding side of the smaller triangle. 17................

[OVER]

Directions (questions 18–23) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

18 If the altitudes of a triangle intersect in a point which is outside the triangle, the triangle is (*a*) acute (*b*) right (*c*) obtuse 18......

19 As the number of sides of a regular polygon increases, each exterior angle of the polygon (*a*) increases (*b*) decreases (*c*) remains the same 19......

20 If in a circle chords AB and CD are perpendicular to each other, then (*a*) arc $AC = 90°$ (*b*) arc AC = arc BD (*c*) the sum of arc AC and arc BD is 180° 20......

21 The locus of the centers of circles tangent to both of two given parallel lines is (*a*) a point (*b*) a line (*c*) two lines 21......

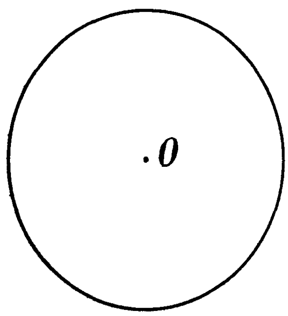
22 In circles O and O', radii OA, OB, $O'A'$, $O'B'$ are drawn making angle AOB = angle $A'O'B'$. If AB and $A'B'$ are drawn, then triangle AOB and triangle $A'O'B'$ *must* be (*a*) congruent (*b*) similar (*c*) equal in area 22......

23 John attends a certain school in which every member of the chess team is a good mathematics student.

Which of the following conclusions is an example of reasoning from a converse? (*a*) John is a good mathematics student. Therefore he is a member of the chess team. (*b*) John is not a good mathematics student. Therefore he is not a member of the chess team. (*c*) John is a member of the chess team. Therefore he is a good mathematics student. 23......

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Inscribe an equilateral triangle in circle O.



25 Divide line segment AB into two segments having the ratio 2:3.