The University of the State of New York

249th High School Examination

PLANE GEOMETRY

Wednesday, August 20, 1930 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1. If two acute angles have their sides respectively perpendicular to each other, the two angles are . . .

2. If the altitude of an equilateral triangle is 15 inches, the medians of the triangle meet in a point . . . inches from each vertex.

3. In the parallelogram ABCD, if BC is less than CD and if the diagonal DB is drawn, then angle ADB is . . . than angle BDC.

4. If similar polygons are constructed on a side and a diagonal of a square, the ratio of their areas is . . .

5. If the sum of the exterior angle at A and the exterior angle at B of the triangle ABC is 270°, then triangle ABC must be a . . . triangle.

6. A diameter of a circle is the locus of the mid-points of a series of . . . chords of the circle.

7. If angle P, formed by two tangents PA and PB drawn to circle O, is 70° and if OA and OB are drawn, then angle AOB contains . . . degrees.

8. The regular polygon whose apothem equals one half a side is called a . . .

9. If the diagonals of an oblique parallelogram form four congruent triangles, the parallelogram is a . . .

10. AB is a diameter of a circle and AC is a chord such that angle BAC is 30°; if AB is 12 inches long, then chord AC is . . . inches long.

11. If two sides of one triangle equal two sides of another triangle and the included angles are supplementary, the two triangles are . . .

12. If one base of a trapezoid is twice the other base and if the altitude is 6 and the area 81, then the shorter base is . . .

13. In the triangle whose sides are 1.5, 2 and 2.5, the angle opposite 2.5 must contain . . . degrees.

14. If the circumference of a circle is 14π, the area is . . . [Leave answer in terms of π.]

15. If two chords intersect within a circle and the ends of the chords are joined by straight lines, either opposite pair of triangles formed are . . .

16. If a central angle and the angle formed by a tangent and a chord intercept the same arc, the ratio of the angles is . . .
17 If the altitude on the hypotenuse of a right triangle divides the hypotenuse into segments 9 and 4, the area of the triangle is ....  
*Ans.* ....

Directions (questions 18–20) — Leave all construction lines on the paper.

18 Find by construction the center of the circle.

![Circle](image)

19 Construct the equilateral triangle whose altitude is the line $AB$.

$A \quad \underline{\quad} \quad B$

20 Divide the line $AB$ into parts proportional to lines $a$, $b$ and $c$.

$\underline{a}$

$\underline{b}$

$\underline{c}$

$A \quad \underline{\quad} \quad B$
Write at top of first page of answer paper (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1930.

The minimum time requirement previous to entering summer high school is five recitations a week for a school year.

For those pupils who have met the time requirement previous to entering summer high school the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1930 is required.

Name the author of the textbook you have used in plane geometry.

PART II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that two triangles are congruent if the three sides of one are equal respectively to the three sides of the other. [12]

22 Prove that the angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [12]

23 In trapezoid $ABCD$, the base $AB$ is twice the base $DC$. If the diagonals $AC$ and $BD$ intersect in point $E$, prove that $CE$ is one third of $AC$. [12]

24 If $X$ is any point in the diagonal $AC$ of parallelogram $ABCD$, prove that triangle $ABX$ is equal in area to triangle $AXD$. [12]

25 By actual construction determine a circle that will be tangent to the sides of a given angle, touching one side at a given point. [12]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of $\pi$ and radicals unless otherwise stated.

26 Two sides of a triangle are 13 inches and 15 inches. If the altitude on the third side is 12 inches, what is the area of the triangle? [12]

27 $ABCDE$ is a regular pentagon inscribed in a circle.
   a Find the number of degrees in the acute angle formed by side $DC$ and the tangent at $D$. [4]
   b Find the number of degrees in the angle formed by extending sides $AB$ and $DC$ to meet. [8]

28 In triangle $ABC$, $AB = 18$, $AC = 12$ and angle $A = 60^\circ$. Find the length of median $AM$ drawn from $A$. [Leave answer in radical form.] [12]
   [Suggestion: Drop perpendiculars to $AB$ from points $C$ and $M$.]

29 A man stands on a 24-foot ladder which touches the wall of a building 16 feet above the ground. If he stands on a round 9 feet from the top of the ladder, how far are his feet from the ground? Find to the nearest foot the distance from the wall to the foot of the ladder. [4].

[2]