PLANE GEOMETRY

June, 1956

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The hypotenuse of a right triangle is 7 and a leg is 4. Find the other leg.

2. The angles of a triangle are in the ratio 3:5:7. Find the number of degrees in the *smallest* angle of the triangle.

3. Two angles are complementary and one angle is 48° greater than the other. Find the number of degrees in the *smaller* angle.

4. Find the number of degrees in the sum of the interior angles of a polygon of twelve sides.

5. Tangents are drawn to a circle from an external point. If the minor arc intercepted by the tangents contains 76°, find the number of degrees in the angle formed by the tangents.

6. The bases of an isosceles trapezoid are 9 and 15 and each base angle contains 45°. Find the altitude of the trapezoid.

7. Find a side of an equilateral triangle whose area is $16\sqrt{3}$.

8. Find the area of a triangle whose base is 12 and whose altitude is 7.

9. Find the area of a circle whose radius is 3.

10. Find the length of an arc of 70° in a circle whose radius is 9.

11. In right triangle ABC, angle $C = 90°$, $AB = 10$ and $AC = 4$. Find angle A to the *nearest degree.*

12. Corresponding sides of two similar triangles are 2 and 3. If the area of the smaller triangle is 12, find the area of the larger triangle.

13. In right triangle ABC, CD is the altitude on the hypotenuse. If $AB = 10$ and $AC = 7$, find AD.

14. Chords AB and CD intersect within a circle at E. If $AE = r$, $EB = s$, and $CE = t$, express ED in terms of r, s and t.

15. From a point outside a circle a tangent and a secant are drawn. If the secant is 12 and its external segment is 3, find the tangent.

16. In parallelogram $ABCD$, DC is extended through C to a point E and AE intersects BC at K. If $DE = 12$, $DC = 8$ and $AK = 10$, find AE.

17. In quadrilateral $ABCD$, it has been proved that AB is parallel to DC and that BC is parallel to AD. Which of the following statements, *a* or *b*, may be used to prove that $ABCD$ is a parallelogram?

a. The opposite sides of a parallelogram are parallel.

b. A parallelogram is a quadrilateral whose opposite sides are parallel.

Directions (18–23): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

18. In circle O, inscribed angle BAC and central angle BOC intercept the same arc BC. If angle $A = 50°$, then angle BOC is equal to (*a*) 25° (*b*) 50° (*c*) 100°

19. In any triangle, the point that is equidistant from the three vertices is the intersection of (*a*) the angle bisectors (*b*) the perpendicular bisectors of the sides (*c*) the medians

20. Quadrilateral $ABCD$ is inscribed in a circle. If angle $A = x°$, then angle C is equal to (*a*) $x°$ (*b*) $x° - 180°$ (*c*) $180° - x°$

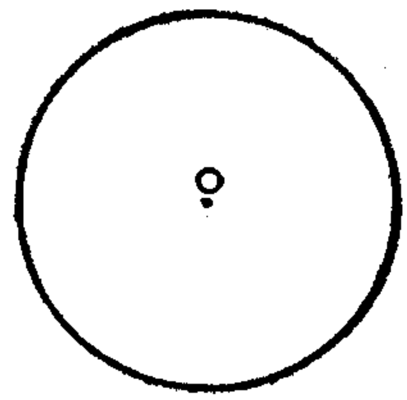
21. The locus of the centers of all circles tangent to two parallel lines is (*a*) a point (*b*) a line (*c*) two lines

22. A regular octagon has a side *s* and an apothem *a*. The area of the octagon is (*a*) $4as$ (*b*) $8as$ (*c*) $16as$

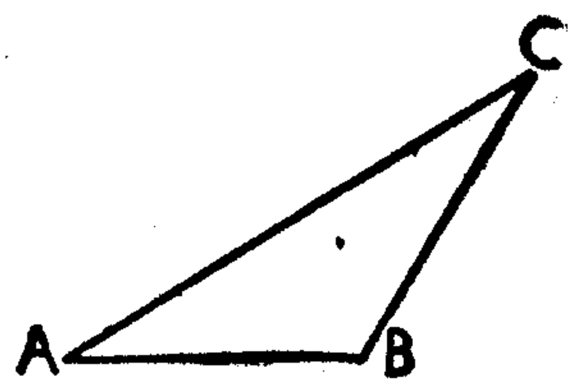
23. The bases of trapezoid *ABCD* are *AD* and *BC*, and its diagonals intersect at *E*. Triangles *AEB* and *CED* are always (*a*) congruent (*b*) similar (*c*) equal in area

Directions (24–25): Leave all construction lines on the paper.

24. Inscribe a square in circle *O*.



25. Construct a triangle congruent to triangle *ABC*.



PART II

Answer three questions from this part.

26. Prove: If two sides of a triangle are equal, the angles opposite these sides are equal. [10]

27. In parallelogram *ABCD*, perpendiculars drawn to diagonal *AC* from *B* and *D* meet *AC* at points *E* and *K* respectively.

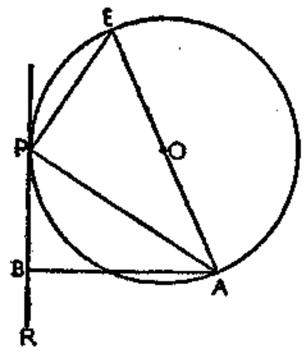
a. Prove: $BE = KD$. [7]

b. A point *H* is taken on *AC*, and *BH* and *DH* are drawn. Prove that triangle *ABH* is equal in area to triangle *ADH*. [3]

28. Prove: If two triangles have the three angles of one equal respectively to the three angles of the other, the triangles are similar. [10]

29. In the figure at the right, PR is a tangent, PA and PE are chords and AE is a diameter of circle O. AB is perpendicular to PR.

Prove: $AB \times AE = (AP)^2$. [10]



30. In triangle ABC, AC is greater than AB. CA is extended through A to a point D, and BD is drawn. Prove that DC is greater than DB. [10]

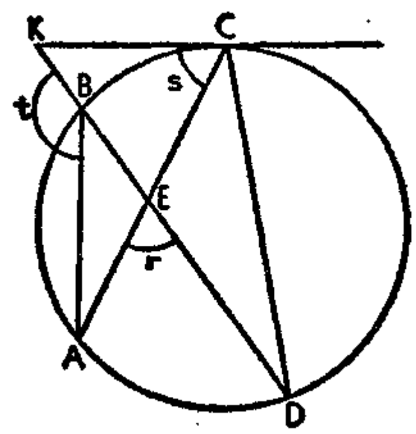
PART III

Answer two questions from this part. Show all work.

31. In the accompanying figure, AB and CD are chords. Chords AC and DB intersect at E. The tangent at C meets DB extended at K. Arc $AB = 80°$ and arcs BC, CD and AD are represented by $x°$, $(2x - 8)°$ and $(x + 32)°$ respectively.

   a. Find the number of degrees in arcs BC, CD and DA. [3, 1, 1]

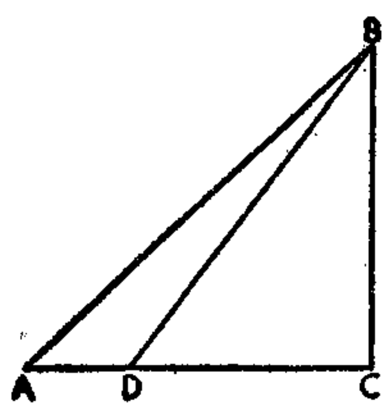
   b. Find the number of degrees in angles r, s and t. [2, 1, 2]



32. In the figure at the right, B represents the position of a captive balloon connected by a cable to a ground station at A. Point C is on the ground directly below the balloon, and D is an observation point. Points A, D and C lie in a straight line on level ground. Angle $A = 43°$, angle $BDC = 54°$, angle $C = 90°$ and $DC = 170$ yards.

   a. Find the height BC of the balloon to the *nearest yard.* [4]

   b. Using the result found in part *a*, find the length AB of the cable to the *nearest yard.* [6]



33. Isosceles triangle ABC, in which $AB = BC$, is inscribed in a circle whose center is O. Altitude BE to base AC is extended to meet the circle at K.

a. If $AC = 8$ and $EK = 2$, find BE. [4]

b. Draw AO and find the ratio of the area of triangle AOB to the area of triangle ABC. [6]

34. In circle O, diameter AB is extended through B to a point P so that $BP = OB$. From point P a tangent is drawn meeting the circle at R, and radius OR is drawn.

a. Find the number of degrees in angle P. [2]

b. If $OR = 6$, find the area of triangle ROP. [Answer may be left in radical form.] [4]

c. Find the area of the figure bounded by PR, PB, and arc RB. [Answer may be left in terms of π and in radical form.] [4]

January, 1957

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find a diagonal of a rectangle whose sides are 3 and 7.

2. The angles of a triangle are in the ratio 1:3:5. Find the number of degrees in the *smallest* angle of the triangle.

3. The area of a rhombus is 48 and one diagonal is 12. Find the other diagonal.

4. In parallelogram $ABCD$, $AB = 12$, $AD = 4$ and angle $A = 30°$. Find the area of the parallelogram.

5. Find the number of degrees in the sum of the interior angles of a polygon of ten sides.