The University of the State of New York
312th HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 18, 1951—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26. Prove that the diagonals of a parallelogram bisect each other. [10]

27. Prove that if two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other. [10]

28. In triangles $ABC$ and $A'B'C'$, side $AB$ equals $A'B'$, side $BC$ equals $B'C'$ and median $CM$ equals median $C'M'$. Prove
   a. triangles $MBC$ and $M'B'C'$ are congruent [6]
   b. triangles $ABC$ and $A'B'C'$ are congruent [4]

29. The bisector of angle $A$ of triangle $ABC$ intersects $BC$ at $D$. Prove
   a. angle $ADC$ is greater than angle $DAC$ [7]
   b. $AC$ is greater than $DC$ [3]

Part III

Answer one question from part III.

30. In the accompanying figure, arc $AB = arc BC = 50^\circ$. Arcs $CD$ and $DA$ are in the ratio 2:3.

   a. Find the number of degrees in arc $CD$ and in arc $DA$. [4]
   b. Chords $AB$ and $DC$ extended intersect at $E$, and chords $AC$ and $BD$ intersect at $F$. Find the number of degrees in angles $AFB$, $BEC$ and $FCE$. [6]
31 Chord $AB$ of a circle is 7.66 inches from the center and arc $AB$ is $80^\circ$. Find
   $a$ the radius of the circle \( [5] \)
   $b$ the area of the circle \( [\text{Use } \pi = 3.14:] \) \( [2] \)
   $c$ the length of the minor arc $AB$ \( [\text{Answer may be left in terms of } \pi.] \) \( [3] \)

**Part IV**

**Answer one question from part IV.**

32 Given trapezoid $ABCD$ with diagonal $AC$ equal to diagonal $BD$. Through $C$ a line is drawn parallel to $BD$ intersecting base $AD$ extended at $E$. Prove

   $a$ $AC = CE$ \( [4] \)
   $b$ triangle $ACD$ is congruent to triangle $ABD$ \( [5] \)
   $c$ $AB = DC$ \( [1] \)

33 Given parallelogram $ABCD$ with $E$ a point on diagonal $AC$. Lines through $E$ intersect $BC$ at $R$, $AD$ at $S$, $AB$ at $T$ and $CD$ at $V$. Prove

   $a$ $AE:EC = TE:EV$ \( [3] \)
   $b$ $AE:EC = SE:ER$ \( [2] \)
   $c$ triangle $TES$ is similar to triangle $REV$ \( [3] \)
   $d$ $TS$ is parallel to $RV$ \( [2] \)
PLANE GEOMETRY

Fill in the following lines:

Name of pupil..............................................Name of school...................................

Part 1

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. The hypotenuse of a right triangle is 17 and one leg is 8. Find the other leg.
2. Express the diagonal of a square in terms of its side $s$.
3. Corresponding sides of two similar polygons are in the ratio 3:5. Find the ratio of their areas.
4. Express the circumference of a circle in terms of its diameter $d$.
5. The diagonals of a rhombus are 6 and 12. Find its area.
6. In parallelogram $ABCD$, side $AB$ is 10, side $AD$ is 8 and angle $A$ is $30^\circ$. Find the area of the parallelogram.
7. The legs of a right triangle are 5 and 8. Find the area of the triangle.
8. Find the area of a trapezoid whose bases are 12 and 20 and whose altitude is 5.
9. The perimeter of a regular polygon is 40 and its apothem is 6. Find the area of the polygon.
10. The sides of a right triangle are 3, 4 and 5. Express as a common fraction the sine of the smallest angle.
11. A tangent and a secant are drawn to a circle from an external point. If the tangent is 12 and the external segment of the secant is 8, find the entire secant.
12. $CD$ is the altitude on the hypotenuse of right triangle $ACB$. If $AD = 9$ and $AB = 13$, find $CD$.
13. $AB$ is tangent to circle $O$ at $B$ and chord $BC$ is drawn forming an angle of $50^\circ$. Find the number of degrees in arc $BC$.
14. Is statement B the converse of statement A? [Answer yes or no.]
   A. If two sides of a triangle are equal, the angles opposite these sides are equal.
   B. The base angles of an isosceles triangle are equal.

Directions (questions 15–18)—If the blank space in each of the following statements is replaced by one of the words always, sometimes or never, the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.

15. If two triangles have a side and any two angles of one equal to the corresponding parts of the other, the triangles are . . . congruent.
16. An altitude of a triangle divides it into two triangles which are . . . similar.

[3]
17 The line joining the mid-points of the legs of a trapezoid divides it into two trapezoids which are \ldots equal.

18 In any circle if two chords are unequal, the longer chord is \ldots nearer the center of the circle.

Directions (questions 19–24) — Indicate the correct answer to each question by writing on the line at the right the letter \textit{a}, \textit{b} or \textit{c}.

19 The locus of points at a given distance from a given straight line is \textit{(a)} a circle \textit{(b)} one straight line \textit{(c)} two parallel lines

20 If a median of a triangle is equal to one half of the side to which it is drawn, the triangle must be \textit{(a)} isosceles \textit{(b)} right \textit{(c)} equiangular

21 If the point of intersection of the altitudes of a triangle is outside the triangle, the triangle is \textit{(a)} acute \textit{(b)} right \textit{(c)} obtuse

22 If the diagonals of a parallelogram are equal, the parallelogram must be a \textit{(a)} rectangle \textit{(b)} square \textit{(c)} rhombus

23 In \textit{a}, \textit{b} and \textit{c} below, the same four terms are arranged in different orders. Which of these orders represents the sequence in which the \textit{definitions} of these four terms should be given?
\begin{itemize}
  \item \textit{a} polygon, triangle, right triangle, hypotenuse
  \item \textit{b} hypotenuse, polygon, right triangle, triangle
  \item \textit{c} triangle, polygon, hypotenuse, right triangle
\end{itemize}

24 To find the center of the circle of which the given arc is a part, the perpendicular bisectors of two chords of the arc are constructed as shown in the accompanying diagram.

Which of the following theorems is the one used to \textit{prove} that the intersection of these bisectors is the required point?
\begin{itemize}
  \item \textit{a} A diameter perpendicular to a chord bisects the chord.
  \item \textit{b} A diameter which bisects a chord, not a diameter, is perpendicular to the chord.
  \item \textit{c} The perpendicular bisector of a chord of a circle passes through the center of the circle.
\end{itemize}

25 Given circle \(O\) and point \(P\) on the circle. Construct a line tangent to the circle at \(P\).