Part II

Answer three questions from part II.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Secants $PAB$ and $PCD$ are drawn to a circle from an external point $P$ and chords $AB$ and $CD$ thus formed are equal. Chord $BD$ is drawn. Prove that
\[ a \ \text{arc} \ AB = \text{arc} \ CD \] [1]
\[ b \ \angle B = \angle D \] [4]
\[ c \ AP = CP \] [5]

28 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [10]

29 $AB$ and $AC$ are tangents to circle $O$ at points $B$ and $C$ respectively. From $C$ a line is drawn perpendicular to diameter $BD$ and intersecting $BD$ at $E$. Lines $AO$, $OC$ and $DC$ are drawn. Prove that
\[ a \ AO \ bisects \ arc \ BC \] [4]
\[ b \ \angle BOA = \angle EDC \] [3]
\[ c \ AB : CE = BO : ED \] [3]
Part III

Answer one question from part III.

30 Angle $B$ of isosceles triangle $ABC$ is $126^\circ$ and side $AB$ is 5 inches.
   a Find, to the nearest tenth of an inch, the length of altitude $BD$. [5]
   b Find, to the nearest tenth of an inch, the length of $AD$. [3]
   c Find, to the nearest square inch, the area of triangle $ABC$. [2]

31 The diagonals of a rhombus are in the ratio 5:12.
   a If the shorter diagonal is represented by $5x$, represent the longer diagonal in terms of $x$. [1]
   b Express the area of the rhombus in terms of $x$. [2]
   c If the area of this rhombus is 120 square inches, find the length of each diagonal. [4]
   d Find one side of the rhombus. [3]

Part IV

Answer one question from part IV.

32 For each of the following statements indicate whether you are given too little information, just enough information, or more information than is needed, to justify the conclusion.
   a If a quadrilateral is inscribed in a circle, the sum of its angles is $360^\circ$. [2]
   b If a diagonal of a quadrilateral divides it into two congruent triangles, the quadrilateral is a parallelogram. [2]
   c If a quadrilateral is circumscribed about a circle, the sum of two opposite sides equals the sum of the other two sides. [2]
   d The line joining the mid-points of the legs of a trapezoid bisects each diagonal. [2]
   e If a quadrilateral is a parallelogram, the figure formed by joining in order the mid-points of its sides is a parallelogram. [2]

33 The bases $AB$ and $CD$ of a trapezoid are 24 and 10 and the legs $AD$ and $BC$ are 13 and 15 respectively. $CE$ is drawn parallel to $DA$, and $CF$ is perpendicular to $AB$ as shown in the figure. Let $EF$ be represented by $x$.
   a Express $FB$ in terms of $x$. [2]
   b Using triangles $EFC$ and $BFC$, write two expressions for $(CF)^2$ in terms of $x$. [2, 2]
   c Find the value of $x$. [2]
   d Find the area of the trapezoid. [2]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Two chords intersecting within a circle intercept opposite arcs of 40° and 80°. Find the number of degrees in the acute angle formed by the chords.

2. A secant and a tangent to a circle from an external point are 8 and 4 respectively. Find the external segment of the secant.

3. The median to the hypotenuse of a right triangle is 5. Find the hypotenuse.

4. A side of a triangle is 12. Find the length of the line segment joining the mid-points of the other two sides.

5. A square is equal in area to a parallelogram whose base is 8 and whose altitude is 2. Find a side of the square.

6. Find the altitude of an equilateral triangle whose side is 5. [Answer may be left in radical form.]

7. A line parallel to side $AB$ of triangle $ABC$ intersects $AC$ at $D$ and $BC$ at $E$. If $DC = 12$, $AD = 4$ and $EC = 18$, find $BE$.

8. Corresponding sides of two similar polygons are in the ratio 3:4. Find the ratio of their areas.

9. An exterior angle of a regular polygon is 40°. Find the number of sides of the polygon.

10. Find the circumference of a circle whose radius is 14. [Use $\pi = \frac{22}{7}$]

11. The angle of a sector of a circle is 60° and the radius of the circle is 6. Find the area of the sector. [Answer may be left in terms of $\pi$.

12. $CD$ is the altitude on the hypotenuse of right triangle $ABC$. $AD = 2$ and $DB = 8$. Find $CD$.

13. $AB$ and $CD$ are two parallel lines 4 inches apart and $P$ is a point on $AB$. How many points are there which are equidistant from $AB$ and $CD$ and 3 inches from $P$?

14. Find the diagonal of a rectangle whose sides are 8 and 15.

15. In parallelogram $ABCD$, angle $A = 30^\circ$. How many degrees are there in angle $B$?

16. The sides of a rectangle are 8 and 10. Find, to the nearest degree, the angle formed by the diagonal and the longer side of the rectangle.

Directions (questions 17–23) — If the blank in each statement is replaced by one of the words always, sometimes, or never, the resulting statement is true. Select the word that will correctly complete each statement and write the word on the line at the right.

17. The diagonals $AC$ and $BD$ of quadrilateral $ABCD$ inscribed in a circle intersect at $E$. Triangle $AED$ is ... similar to triangle $BEC$.

18. From external point $A$ tangents $AB$ and $AC$ are drawn to a circle and chord $BC$ is drawn. Triangle $ABC$ is ... equilateral.

19. The locus of the center of a circle of given radius and tangent externally to a given circle is ... a circle.
20 The areas of two triangles having equal altitudes ... are to each other as their bases.

21 The medians of a triangle ... bisect each other.

22 If angle $A$ of triangle $ABC$ is greater than angle $A'$ of triangle $A'B'C'$, then $BC$ is ... greater than $B'C'$.

23 It is ... possible to construct triangle $ABC$ if the given parts are side $AB$, angle $A$ and the altitude on $AB$.

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct a circle which is tangent to line $AB$ at point $O$ and whose center is on line $CD$.

25 Divide line segment $RS$ into segments having the ratio $a:b$. 

\[ \frac{a}{b} \]