

The University of the State of New York

303D HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, June 21, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

- 26 Prove that tangents drawn to a circle from an external point are equal. [10]
- 27 In a circle whose center is O , AB is a diameter and AE a chord. From any point C on chord AE , CD is drawn perpendicular to AB . Prove that $AB \times AD = AC \times AE$. [10]
- 28 CD is the median to hypotenuse AB of right triangle ABC . If CD is extended its own length to E and EA and EB are drawn, prove that $AEBC$ is a rectangle. [10]
- 29 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

Part III

Answer one question from part III.

- 30 The area of a parallelogram is 60 sq. in. The longer side is 12 in. and one of the angles of the parallelogram is 54° . Find to the nearest tenth of an inch the shorter side of the parallelogram. [10]
- 31 A circle is inscribed in an equilateral triangle whose side is 6. Find to the nearest integer the difference between the area of the triangle and the area of the circle. [$\pi = 3.14$ and $\sqrt{3} = 1.73$] [10]

Part IV

Answer one question from part IV.

- 32 a Prove that any point on the bisector of an angle is equidistant from the sides of the angle. [4]
- b State and prove the converse of a. [2, 3]
- c Is either the theorem in a or the converse stated in answer to b, taken alone, sufficient for proving the statement: The locus of points equidistant from the sides of an angle is the angle bisector? [Answer yes or no.] [1]

[1]

[OVER]

33 Each of the five parts of this question is a statement that can be correctly completed by *two* and *only two* of the given choices. Write the numbers 1 to 5 on your answer paper and after *each* indicate the correct answer to the corresponding question by writing only *two* of the letters *a, b, c, d*.

(1) In triangle ABC , E is the mid-point of AC , D the mid-point of BC and ED is drawn; then
 (a) $AE:AC = BD:BC$ (b) $\triangle CED = \frac{1}{2} \triangle ABC$ (c) $ED = \frac{1}{2} AB$ (d) $BD = AE$ [2]

(2) If the perpendicular bisectors of the sides of a triangle meet in a point outside the triangle, the triangle (a) is acute (b) is right (c) is obtuse (d) can be isosceles [2]

(3) If a triangle is inscribed in a circle and one angle is double another, (a) the triangle must be a $30^\circ-60^\circ$ right triangle (b) the triangle may be isosceles (c) the arcs intercepted by these angles are in the ratio 2:1 (d) the sides opposite these angles are in the ratio 2:1 [2]

(4) In triangle ABC , if angle A is less than 60° , (a) AC can be the shortest side of triangle ABC (b) BC can be the longest side of triangle ABC (c) BC can be equal to AB (d) AB , AC and BC can be equal to each other [2]

(5) If the diagonals of rhombus $ABCD$ intersect at E , then (a) $\tan \frac{1}{2}A = \frac{EB}{EA}$ (b) the area of the rhombus is $AE \times DB$ (c) the area of the rhombus is $AB \times AD$ (d) a circle can be circumscribed about the rhombus [2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 The hypotenuse of a right triangle is 17 and one leg is 15. Find the other leg. 1.....
- 2 The hypotenuse AB of right triangle ABC is twice leg BC . Find the number of degrees in angle ABC . 2.....
- 3 In right triangle ABC , altitude CD is drawn on the hypotenuse. If $CD = 6$ and $AD = 3$, find DB . 3.....
- 4 Find the altitude of an equilateral triangle whose side is 4. [Answer may be left in radical form.] 4.....
- 5 In triangle ABC , angle $C = 90^\circ$, $\tan A = .3$ and $AC = 20$. Find BC . 5.....
- 6 How many degrees are there in each angle of a regular polygon of 10 sides? 6.....
- 7 A trapezoid is inscribed in a circle. The arcs determined by the bases are 50° and 160° . How many degrees are there in one of the arcs intercepted between the bases? 7.....
- 8 From point P outside a circle, tangent PD and secant PCA are drawn. If secant $PCA = 16$ and external segment $PC = 4$, find tangent PD . 8.....
- 9 An angle formed by a tangent and a secant is 75° . If the greater intercepted arc is 180° , find the number of degrees in the smaller intercepted arc. 9.....
- 10 Find the area of a trapezoid if the altitude is 10 and the bases are 6 and 8. 10.....
- 11 Find the side of a square that is equal in area to a triangle whose base is 36 and whose altitude is 8. 11.....
- 12 Corresponding sides of two similar triangles are in the ratio 2:3. Find the ratio of the area of the smaller triangle to the area of the larger triangle. 12.....
- 13 Two similar polygons have 2 and 3 as a pair of corresponding sides. Find the ratio of the perimeter of the smaller polygon to the perimeter of the larger polygon. 13.....
- 14 The area of a sector of a circle is one fifth the area of the circle. How many degrees are there in the angle of the sector? 14.....
- 15 Find the length of an arc of a circle if the arc is 30° and the radius is 12. [Answer may be left in terms of π .] 15.....

Directions (questions 16-18) — Indicate the correct answer to each question by writing on the line at the right the letter a , b or c .

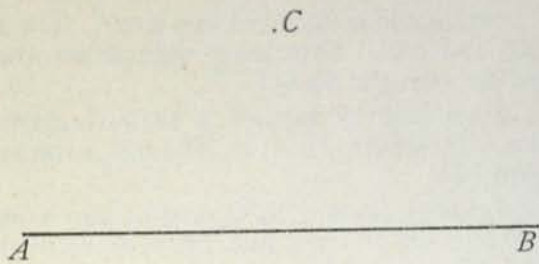
- 16 In triangle ABC , angle A is greater than angle B and the bisectors of angle A and angle B meet in D . Then BD is (a) greater than AD (b) equal to AD (c) less than AD 16.....
- 17 The locus of points at a given distance from a given line is (a) one line (b) two lines (c) a circle 17.....
- 18 If the sum of two exterior angles of a triangle is 270° , the triangle is (a) acute (b) right (c) obtuse 18.....

Directions (questions 19-23) — In each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

- 19 The area of a regular polygon is equal to one half the product of its perimeter and its apothem. 19.....
- 20 If chord AB of a circle bisects chord CD , then AB is perpendicular to CD . 20.....
- 21 The altitudes of a triangle meet at a point that is inside the triangle. 21.....
- 22 A median of an equilateral triangle is equal to an altitude of the triangle. 22.....
- 23 The diagonals of a rhombus are equal. 23.....

Directions (questions 24-25) — Leave all construction lines on your paper.

- 24 From point C , construct a line perpendicular to AB .



- 25 Divide line segment AB into three equal parts.

