The University of the State of New York

291ST HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 20, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Part II

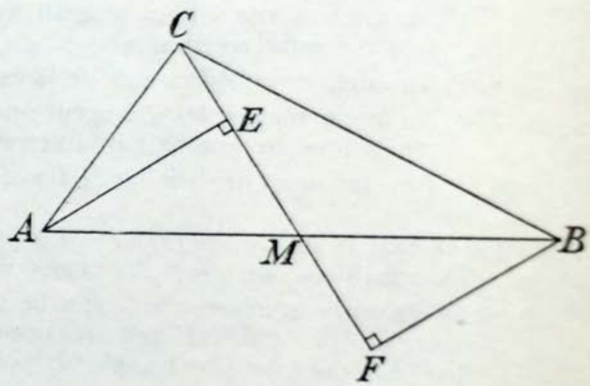
Answer two questions from part II.

26 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the two intercepted arcs. [10]

27 *a* For the following exercise, draw a figure, letter it and, in terms of the figure, state what is given and what is to be proved: [No proof is required in this part.]

If two sides of a triangle and the altitude drawn to one of these sides are proportional to the corresponding sides and altitude of another triangle, the two triangles are similar. [3]

b In the figure at the right, CM is the median to side AB of triangle ABC. AE is perpendicular to CM and BF is perpendicular to CM extended. If AF and BE are drawn, prove that $AFBE$ is a parallelogram. [7]



28 *a* Prove the theorem: Any point that is equidistant from the ends of a line segment lies on the perpendicular bisector of the segment. [5]

b State the converse of the theorem given in *a*. [3]

c Is either the theorem in *a* or the theorem stated in answer to *b* alone sufficient to prove the following theorem: The perpendicular bisector of a line segment is the locus of all points equidistant from the ends of the segment? [2]

Part III

Answer two questions from part III.

29 Bases *AB* and *DC* of trapezoid *ABCD* are 30 and 20 respectively and its legs *AD* and *BC* are each 13. *AD* and *BC* are extended to meet at point *E*.

a Find the altitude of the trapezoid. [3]

b Find the altitude of triangle *ABE*. [5]

c Find the area of triangle *ABE*. [2]

30 [In this exercise use $\pi = 3.14$, $\sqrt{3} = 1.73$ and express each result correct to the *nearest tenth*.] In circle *O* chord *AB* is equal to a radius and each is 6 inches in length. If radii *OA* and *OB* are drawn, find

a The length of minor arc *AB* [3]

b The area of triangle *AOB* [3]

c The area of sector *AOB* [3]

d The area of the minor segment of the circle [1]

31 The perimeter of a regular polygon of 10 sides is 100 inches.

a Find the length of the apothem of the polygon correct to the *nearest tenth of an inch.* [7]

b Using the result found in answer to *a*, find the area of the polygon. [3]

Part IV

Answer one question from part IV.

32 *a* Write a brief explanation of the term *circular reasoning* as applied to geometry. [6]

b Two of the propositions below are not used in proving the proposition: If a tangent and a secant are drawn from an outside point to a circle, the tangent is the mean proportional between the secant and its external segment. Indicate which *two* of the following are *not* used: [4]

(1) Two triangles are similar if two angles of one triangle are equal respectively to two angles of the other.

(2) An angle formed by a tangent and a secant is measured by one half the difference of the intercepted arcs.

(3) An angle inscribed in a circle is measured by one half its intercepted arc.

(4) An angle formed by a tangent and a chord drawn from the point of contact is measured by one half its intercepted arc.

(5) Two triangles are similar if the corresponding sides are in proportion.

33 *a* Prove that if the adjacent sides of a parallelogram are unequal, a diagonal of the parallelogram does not bisect the angles whose vertices it joins. [6]

b Show how the statement in *a* can be used to prove the following exercise: Given the triangle *ABC* with *AB* and *AC* unequal and with *AM* the median to side *BC*; prove that *AM* does not bisect angle *A*. [4]

Fill in the following lines:

Name of school..Name of pupil..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If an acute angle of a parallelogram contains 73°, how many degrees are there in an obtuse angle of the parallelogram? 1................

2 Find an altitude of an equilateral triangle whose side is 10. [Answer may be left in radical form.] 2................

3 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 2 and 8. Find the altitude. 3................

4 The area of a parallelogram is 30 and the base is 10. Find the altitude drawn to the base. 4................

5 If the sum of the angles of a polygon is 540°, how many sides has the polygon? 5................

6 Find a side of the rhombus whose diagonals are 8 and 6. 6................

7 The line segment which joins the mid-points of two sides of a triangle is 3. Find the third side. 7................

8 The altitude of a trapezoid is 6 and its bases are 8 and 12. Find the area of the trapezoid. 8................

9 Two secants from an external point P intercept arcs of 90° and 20° on a circle. Find the number of degrees in angle P. 9................

10 Triangle ABC is inscribed in a circle. If arc $AB = 64°$ and arc $BC = 94°$, find the number of degrees in angle B. 10................

11 Tangents PA and PB from an external point P to circle O form an angle of 70°. If radii OA and OB are drawn, how many degrees are there in angle AOB? 11................

12 In triangle ABC, angle $C = 90°$, $AB = 30$ and $BC = 15$. How many degrees are there in angle A? 12................

13 A diameter of a circle is perpendicular to a chord. The length of the chord is 12 inches and the length of the shorter segment of the diameter is 3 inches. Find the number of inches in the length of the longer segment of the diameter. 13................

14 In a circle, two parallel chords on opposite sides of the center have arcs of 100° and 120°. Find the number of degrees in one of the arcs included between the chords. 14................

15 The areas of two similar triangles are in the ratio 9:1. A side of the larger triangle is 12. What is the corresponding side of the smaller triangle? 15................

16 Two points A and B are 6 inches apart. How many points are there that are equidistant from both A and B and also 5 inches from A? 16................

Directions (questions 17–23) — If the blank in each statement is filled by one of the words *always, sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

17 The diagonals of a parallelogram are ... equal. 17................

18 As the number of sides of a regular polygon inscribed in a circle increases, the apothem ... increases. 18................

19 If any two sides of a right triangle are equal to the corresponding sides of another right triangle, the triangles are . . . congruent. 19...............

20 If one of the equal sides of an isosceles triangle is greater than the base, then the angle opposite the base is . . . greater than 60°. 20...............

21 If two circles are concentric, any two chords of the larger circle which are tangent to the smaller circle are . . . equal. 21...............

22 If an angle of one isosceles triangle is equal to the corresponding angle of another isosceles triangle, the two triangles are . . . similar. 22...............

23 The perpendicular bisectors of sides AB and BC of triangle ABC intersect at O. The perpendicular bisector of AC . . . passes through O. 23...............

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct an isosceles triangle whose base is the line segment b and whose altitude upon the base is the line segment h.

h

b

25 Divide line segment AB into parts that shall be proportional to the two given segments m and n.

A B

m

n