Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for a school year.
Fill in the following lines:

Name of school.............................................Name of pupil..............................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–8) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The angle formed by two secants intersecting outside a circle is measured by one half the ... of the intercepted arcs.

2 The area of a circle is $25\pi$ square inches. The radius of this circle is ... inches.

3 If the vertex angle of an isosceles triangle is $40^\circ$, the number of degrees in either of the exterior angles formed by extending the base is ... .

4 If each exterior angle of a regular polygon contains $45^\circ$, the number of sides of the polygon is ... .

5 In triangle $ABC$ angle $B = 30^\circ$ and side $AB = 6$ inches. The length of the altitude $AD$ upon side $BC$ is ... inches.

6 A pair of corresponding altitudes of two similar triangles are 4 inches and 2 inches. The area of the larger triangle is ... times the area of the smaller.

7 The radius of a circle is 13 inches and a chord of this circle is 10 inches. The distance of this chord from the center of the circle is ... inches.

8 At a point 100 feet from the foot of a flagpole the angle of elevation of the top of the pole is $31^\circ$. The height of the flagpole, correct to the nearest foot, is ... feet.

Directions (questions 9–12) — Indicate the correct answer to each of the following questions by writing the letter $a$ or $b$ on the line at the right.

9 The area of a rhombus is equal to ($a$) the product of the two diagonals or ($b$) one half the product of the two diagonals.

10 An axiom is a statement which ($a$) is to be proved or ($b$) is accepted without proof.

11 The circumference of a circle whose radius is 4, is ($a$) $8\pi$ or ($b$) $16\pi$.

12 The locus of the centers of all circles tangent to each of two parallel lines is ($a$) one straight line or ($b$) two straight lines.
Directions (questions 13–17) — Indicate whether each of the following statements is always true, sometimes true or never true by writing the word always, sometimes or never on the dotted line at the right.

13. When two straight lines are cut by a transversal, if the two interior angles on the same side of the transversal are supplementary, the two straight lines are parallel. 13

14. If two angles and a side of one triangle are equal to the corresponding parts of another triangle, then the triangles are congruent. 14

15. A triangle can be constructed having sides 8 inches, 12 inches and 3 inches. 15

16. If a theorem is true, a converse of the theorem is true. 16

17. A median of a triangle divides it into two triangles which are equal in area. 17

Directions (questions 18–20) — Leave all construction lines on the paper.

18. Construct the bisector of angle $ABC$.

19. Inscribe an equilateral triangle in circle $O$.

20. Find by construction the two points which are equidistant from points $A$ and $B$ and also at the given distance $s$ from point $C$. 20
21 Prove that the diagonals of a parallelogram bisect each other. [10]

22 Prove that if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the secant and its external segment. [10]

23 Given triangle \( ABC \) with \( AB = AC \); \( D \) is any point between \( B \) and \( C \) in \( BC \) and line segment \( AD \) is drawn. Prove that \( AB > AD \). [10]

24 \( PA \) and \( PB \) are tangents drawn to a circle whose center is \( O \), \( A \) and \( B \) being the points of tangency. Line segment \( PO \) is drawn and extended to meet the circle at \( C \). Prove that arc \( AC = \text{arc} \ BC \). [The use of original exercises as reasons will not be allowed in this proof.] [10]

25 In the figure at the right, \( AB \) is the diameter of a semicircle. A point \( P \) moves along the arc from \( A \) to \( B \) and chords \( PA \) and \( PB \) are drawn.

a Using the expression \( \text{increases, decreases or remains the same,} \) indicate the change, if any, that takes place in (1) angle \( PAB \), (2) angle \( APB \) and (3) angle \( PBA \). [6]

b At what position of \( P \) will the altitude of triangle \( APB \) on base \( AB \) be greatest? [2]

c At what position of \( P \) will the area of triangle \( APB \) be greatest? [2]

Group III

Answer two questions from this group.

26 In the drawing at the right, \( ABCD \) is a quadrilateral inscribed in a circle; arc \( AB = 135^\circ \), arc \( BC = 40^\circ \) and arc \( CD = 100^\circ \). Chords \( AC \) and \( BD \) are drawn; also chords \( AB \) and \( DC \) are extended to meet at \( E \) and the tangent at \( A \) meets \( CD \) extended at \( F \). Find the number of degrees in angle \( a \), angle \( b \), angle \( c \), angle \( d \) and angle \( e \). [10]

27 Given two equal circles with a square inscribed in one and a regular hexagon in the other; if the radius of each circle is 14 inches, find, correct to the nearest tenth of a square inch, the difference in the areas of the regular hexagon and the square. [Use \( \sqrt{3} = 1.73 \)] [10]

28 The bases of an isosceles trapezoid are 8 and 28. One base angle is \( 53^\circ \).

a Find, correct to the nearest tenth, the altitude of the trapezoid. [7]

b Find, correct to the nearest integer, the area of the trapezoid. [3]