The University of the State of New York

245th High School Examination

PLANE GEOMETRY

Wednesday, June 19, 1929—9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Fill in the following lines:

Name of school..............................................Name of pupil..............................................

Detach this sheet and hand it in at the close of the one hour period.

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 A quadrilateral is a parallelogram if the opposite sides are ......... Ans..............................................

2 If $AB$ is parallel to $CD$ and if angle $a$ is twice angle $b$, then the number of degrees in angle $c$ is ......

3 In triangle $ABC$, $AC = BC$. If angle $C = 40^\circ$, then the exterior angle at $A = ......

4 In triangle $ABC$, if side $AB$ is greater than side $AC$ and if angle $B = 60^\circ$, then angle $A$ is .... than angle $B$.

5 $AB$ is a diameter of circle $O$ whose radius is $10^\circ$. At point $C$ on $AB$ $8^\circ$ from $O$, a perpendicular is drawn to $AB$ terminating in the circle. The length of this perpendicular is ......

6 If chord $CD$ is the perpendicular bisector of chord $AB$, then $CD$ is a ......

7 $PA$ and $PB$ are tangents to circle $O$ from point $P$. If angle $APB = 60^\circ$ and $PA = 8$, the length of chord $AB$ is ......

8 Two chords, $AB$ and $AC$, and two radii, $OB$ and $OC$, intercept the same arc $BC$. If angle $A = 33^\circ$, then angle $COB = ......

9 In a circle whose radius is 10, a central angle of $120^\circ$ intercepts an arc whose length in terms of $\pi$ is ......

10 If one side of an equilateral triangle is $a$, then the altitude in terms of $a$ is ......

11 A line $3^\circ$ long joins the mid-points of two sides of a triangle; the length of the third side is ......

12 Finding a point in a triangle equidistant from the three vertices involves bisecting two ......

[OVER]
13 A point \( D \) is 3" from the center of a circle whose radius is 5". The product of the segments of any chord through \( D \) is ....

14 The area of a right triangle whose sides are 6, 8 and 10 is ....

15 If one side of a regular hexagon is 6, its area expressed in radical form is ....

16 The corresponding bases of two similar triangles are 3" and 5". The ratio of their areas is ....

17 One angle of a right triangle is 60°; if the hypotenuse is 10, the shortest side is ....

Directions (questions 18-20) — Leave all construction lines on the paper.

18 Divide the line \( AB \) into three equal parts.

\[ \text{A} \quad \overline{\text{A} \text{B}} \quad \text{B} \]

19 Transform the parallelogram \( ABCD \) into a triangle.

\[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D} \\
\text{A} \\
\text{B} \\
\end{array} \]

20 On line \( DE \) corresponding to side \( AB \), construct a triangle similar to triangle \( ABC \).

\[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D} \\
\text{E} \\
\end{array} \]
PLANE GEOMETRY

Wednesday, June 19, 1929

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

PART II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [12]

22 $ADE$ and $BCF$ are equilateral triangles constructed on sides $AD$ and $BC$ of square $ABCD$ outside the square. If lines $EB$ and $DF$ are drawn, prove that $ED = BF$ and $EB = DF$. [3, 9]

23 Through any point $A$ on a circle, a tangent is drawn and two unequal chords $AB$ and $AC$. A chord parallel to the tangent cuts $AB$ and $AC$ in $D$ and $E$ respectively. Prove that $AB : AE = AC : AD$. [12]

24 Diagonal $AC$ of quadrilateral $ABCD$ divides the figure into two triangles equal in area but not congruent.

a) Prove that points $D$ and $B$ are equidistant from $AC$. [6]

b) If diagonal $BD$ is drawn, prove that $AC$ bisects $BD$. [6]

25 $ABC$ is an isosceles triangle and the bisectors of the equal angles $A$ and $B$ meet in $O$. Prove that $AB$ is greater than $AO$. [12]

Group II

Answer two questions from this group.

Irrational results may be left in the form of $\pi$ and radicals unless otherwise stated.

26 $ABCD$ is a trapezoid with bases $AB$ and $DC$. Diagonals $AC$ and $BD$ intersect in $O$. If $AB = 20$, $DC = 4$ and the altitude of $ABCD = 6$, find the length of the perpendicular from $O$ to $AB$. [12]

27 $ABC$ is a triangle circumscribing a circle $O$. Angle $A = 60^\circ$ and angle $B = 40^\circ$. Find the angles of the triangle formed by joining the points of tangency. [12]

28 $ABC$ is a right triangle with the right angle at $C$. On $AB$ an equilateral triangle $DAB$ is constructed outside the given triangle, and line $DC$ is drawn. If angle $ABC = 30^\circ$ and $AB = 4$, find the length of $CD$. [Leave answer in radical form.] [12]

29 A wheel has been broken so that only a portion of the rim remains. In order to find the radius of the wheel the following measurements are made: Three points $A$, $B$ and $C$ are marked on the rim so that chord $AB =$ chord $AC$. The chords are then measured, and $AB = 15''$, $AC = 15''$, $BC = 24''$. Find the radius of the wheel and the length of the rim to the nearest inch. [Use $\pi = 3.14$] [12]