

PLANE GEOMETRY

Wednesday, June 15, 1927 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than three from group I and at least one from group II.

Group I

Do not answer more than three questions from this group.

1 Prove that when two straight lines are cut by a third, if the alternate interior angles are equal, the two straight lines are parallel. $[12\frac{1}{2}]$

2 Prove that the tangents drawn to a circle from an external point are equal. $[12\frac{1}{2}]$

3 Prove that if two triangles have the angles of one respectively equal to the angles of the other, the triangles are similar. $[12\frac{1}{2}]$

4 Prove that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. $[12\frac{1}{2}]$

Group II

Answer at least one question from this group.

Problems in this group should be constructed accurately with ruler and compasses. Leave all construction lines on the paper.

5 Inscribe an equilateral triangle in a given circle. $[12\frac{1}{2}]$

6 Given two parallel lines.

a Construct a parallelogram having its base AB in one of the given lines, the opposite side CD in the other given line and the diagonals intersecting in O . [3]

b A number of parallelograms are constructed as in answer to a, each having the same line AB for its base; what is the locus of point O for these parallelograms? $[9\frac{1}{2}]$

Group III

Irrational results may be left in the form of π and radicals unless otherwise stated.

7 ABC is any triangle and angle A is bisected by a line meeting BC in D ; prove that AB is longer than BD . $[12\frac{1}{2}]$

8 In a circle the length of an arc of 72° is 6 inches; find the area of the circle. $[12\frac{1}{2}]$

9 In the parallelogram $ABCD$, angle $B = 120^\circ$, $AB = 20$ and $BC = 12$. Find the length of the altitude on AB drawn from C and the length of diagonal AC . $[12\frac{1}{2}]$

10 a Prove that if the altitude on a side of a triangle is the mean proportional between the segments of that side, then the two triangles formed by this altitude are similar. [9]

b State the converse of a. [Proof not required in b] $[3\frac{1}{2}]$

11 A line 6 inches long is drawn parallel to side AB of triangle ABC and terminated by sides AC and BC . This line cuts off a triangle whose area is $\frac{1}{4}$ of triangle ABC . Find the length of base AB . $[12\frac{1}{2}]$

12 ABC is an inscribed equilateral triangle. R and S are the mid-points of arcs AC and BC respectively. Line RS intersects side AC at D and side BC at E .

a Prove $DR = DC$. [6]

b Prove $DC = DE$. $[6\frac{1}{2}]$

13 A railway curve is in the form of an arc of a circle. Two points A and B are taken on this curve. The length of chord AB is found to be 120 feet and the distance from the mid-point of this chord to the curve is found to be 12 feet. What is the radius of the circle? $[12\frac{1}{2}]$