

## PLANE GEOMETRY

Wednesday, June 16, 1926—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

*Answer eight questions, including not more than three from group I and at least one from group II.*

## Group I

Do not answer more than three questions from this group.

1 Prove that if in the same circle or in equal circles two chords are equally distant from the center, they are equal.  $[12\frac{1}{2}]$

2 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar.  $[12\frac{1}{2}]$

3 Prove that the area of a parallelogram is equal to the product of its base and its altitude.  $[12\frac{1}{2}]$

4 Prove that if two triangles have two sides of one respectively equal to two sides of the other and the included angles unequal, the triangle which has the greater included angle has the greater third side.  $[12\frac{1}{2}]$

## Group II

Answer at least one question from this group.

*Problems in this group should be constructed accurately with ruler and compasses. Leave all construction lines on the paper.*

5 a Given three points not in a straight line; construct a circle that will pass through these three points. [6]

b Divide a given line  $l$  into three equal parts.  $[6\frac{1}{2}]$

6 Given circle  $O$  and a fixed secant through its center; locate all points outside of circle  $O$  that are a given distance  $d$  from the secant and also from the circle.  $[12\frac{1}{2}]$

## Group III

*Irrational results may be left in the form of  $\pi$  and radicals unless otherwise stated.*

7 A circle and a square each have a perimeter of  $220'$ ; find the area of each.  $[8, 4\frac{1}{2}]$

8 Two vertical poles  $48'$  apart are located in a level field. A wire is stretched taut (tight) between the tops of the two poles. The poles extend  $24'$  and  $44'$  respectively above the ground. Find the length of the wire, allowing  $1'$  at each end for fastening.  $[12\frac{1}{2}]$

9 a Prove that if one angle of a triangle is a right angle, the median to the opposite side is equal to one half of that side.  $[9\frac{1}{2}]$

b State the converse of a. [Proof not required in b.] [3]

10 Given two circles with radii  $15''$  and  $9''$  and a common internal tangent whose length is  $32''$ ; find (a) the segments into which the tangent is divided by the line of centers, (b) the distance between the two centers.  $[6, 6\frac{1}{2}]$

11  $ABCD$  is an inscribed quadrilateral with  $AB$  and  $DC$  extended through  $B$  and  $C$  respectively to meet in point  $E$ . Diagonals  $AC$  and  $BD$  meet in  $P$ . Angle  $E$  is  $20^\circ$  and angle  $APD$  is  $80^\circ$ . Find the number of degrees (a) in each of the two arcs  $AD$  and  $BC$ , (b) in angle  $ACD$ .  $[10, 2\frac{1}{2}]$

12 In quadrilateral  $ABCD$  angle  $B$  and angle  $C$  are two equal obtuse angles and side  $AB$  is less than side  $DC$ ; if  $AB$  and  $DC$ , produced through  $B$  and  $C$  respectively, meet in point  $E$ , prove that angle  $ADE$  is less than angle  $EAD$ .  $[12\frac{1}{2}]$

13  $ABCDEF$  is a regular inscribed hexagon.  $BD$  meets  $AC$  in  $K$  and  $EC$  in  $M$ . Draw  $AE$ . Prove that (a) triangles  $KCM$  and  $ACE$  are equilateral, (b)  $BK = KM = MD$ , (c) triangle  $KCM = \frac{1}{3}$  of triangle  $ACE$ .  $[4\frac{1}{2}, 4, 4]$