PLANE GEOMETRY

Wednesday, June 16, 1926 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper: (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than three from group I and at least one from group II.

Group I

Do not answer more than three questions from this group.

1. Prove that if in the same circle or in equal circles two chords are equally distant from the center, they are equal. [12½]

2. Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [12½]

3. Prove that the area of a parallelogram is equal to the product of its base and its altitude. [12½]

4. Prove that if two triangles have two sides of one respectively equal to two sides of the other and the included angles unequal, the triangle which has the greater included angle has the greater third side. [12½]

Group II

Answer at least one question from this group.

Problems in this group should be constructed accurately with ruler and compasses. Leave all construction lines on the paper.

5. a. Given three points not in a straight line, construct a circle that will pass through these three points. [6]

   b. Divide a given line into three equal parts. [6½]

6. Given circle O and a fixed secant through its center, locate all points outside of circle O that are a given distance d from the secant and also from the circle. [12½]

7. A circle and a square each have a perimeter of 220'; find the area of each. [8, 4½]

8. Two vertical poles 48' apart are located in a level field. A wire is stretched taut (tight) between the tops of the two poles. The poles extend 24' and 44' respectively above the ground. Find the length of the wire, allowing 1' at each end for fastening. [12½]

9. a. Prove that if one angle of a triangle is a right angle, the median to the opposite side is equal to one half of that side. [9½]

   b. State the converse of a. [Proof not required in b.] [3]

10. Given two circles with radii 15" and 9" and a common internal tangent whose length is 32"; find (a) the segments into which the tangent is divided by the line of centers, (b) the distance between the two centers. [6, 6½]

11. ABCD is an inscribed quadrilateral with AB and DC extended through B and C respectively to meet in point E. Diagonals AC and BD meet in P. Angle E is 50° and angle APD is 80°. Find the number of degrees (a) in each of the two arcs AD and BC, (b) in angle ACD. [10, 2½]

12. In quadrilateral ABCD angle B and angle C are two equal obtuse angles and side AB is less than side DC; if AB and DC, produced through B and C respectively, meet in point E, prove that angle ADE is less than angle EAD. [12½]

13. ABCDEF is a regular inscribed hexagon. BD meets AC in K and EC in M. Draw AE. Prove that (a) triangles KCM and ACE are equilateral, (b) BK = KM = MD, (c) triangle KCM = 1/3 of triangle ACE. [4½, 4, 4]