

PLANE GEOMETRY

Wednesday, June 20, 1923—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than three from group I and at least one from group II.

Group I

Do not answer more than three questions from this group.

- 1 Prove that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram. $[12\frac{1}{2}]$
- 2 Prove that an angle formed by a tangent and a chord through the point of contact, is measured by one half the intercepted arc. $[12\frac{1}{2}]$
- 3 Prove that if in a right triangle a perpendicular is drawn from the vertex of the right angle to the hypotenuse,
 - a the two triangles thus formed are similar to the given triangle and similar to each other; $[8]$
 - b the perpendicular is the mean proportional between the segments of the hypotenuse. $[4\frac{1}{2}]$
- 4 Prove that the areas of two similar polygons are to each other as the squares of any two corresponding sides. $[12\frac{1}{2}]$

Group II

Answer at least one question from this group.

Leave all construction lines on the paper.

- 5 Divide a given line a into two parts proportional to two given unequal lines m and n . $[12\frac{1}{2}]$
- 6 Construct a right triangle, using a given line as the hypotenuse and another given line as a leg. $[12\frac{1}{2}]$

Group III

Irrational results may be left in the form of π and radicals unless otherwise stated.

- 7 Prove that if the upper base of an isosceles trapezoid is equal to either leg, the diagonals bisect the angles at the ends of the lower base. $[12\frac{1}{2}]$
- 8 The base AB of a scalene triangle ABC is divided into four equal parts by points D, E, F taken in order. Show that the line from D to the center of AC equals the line from F to the center of BC . $[12\frac{1}{2}]$
- 9 How far from a circle whose radius is 6 must a point be selected so that the whole secant from that point through the center of the circle shall be twice the tangent from that point? $[12\frac{1}{2}]$
- 10 An equilateral triangle inscribed in a given circle has an area equal to $144\sqrt{3}$. What is the area of the circle? $[12\frac{1}{2}]$
- 11 Show how to find a point that is equidistant from two given points and also equidistant from two given parallel lines. $[12\frac{1}{2}]$