The University of the State of New York

213TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, June 15, 1915—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in your study of plane geometry.

Answer eight questions, including question 12.

1 Prove that if two triangles have two sides of the one respectively equal to two sides of the other, and the included angles unequal, the triangle which has the greater included angle has the greater third side. [12]

2 Prove that if in the same circle, or in equal circles, two chords are equal, they are equally distant from the center. [12]

3 Prove that the area of a trapezoid is equal to the product of its altitude and half the sum of its parallel sides. [12]

4 Prove that if a circle is divided into any number of equal parts (*a*) the chords forming the successive points of division form a regular polygon inscribed in the circle, (*b*) the tangents drawn at the points of division form a regular polygon circumscribed about the circle. [12]

5 Prove that the homologous medians of two similar triangles have the same ratio as any two homologous sides. [12]

6 *A*, *B*, *C* are three points on a circle. The bisector of angle *ABC* meets the circle again at *D*. *DE* is drawn parallel to *AB* and meets the circle again at *E*. Prove that *DE*=*BC*. [12]

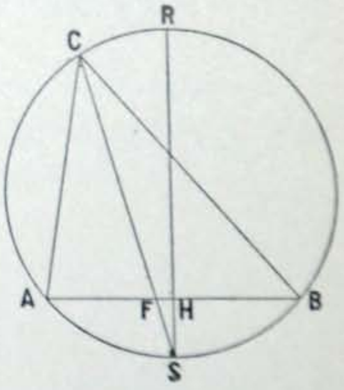
7 The triangle *ABC* is inscribed. *CF* bisects the angle *ACB*. *RS* is the perpendicular bisector of *AB*.

Why does *CF* pass through *S*? [3]

Why does *RS* pass through the center of the circle? [3]

Why is *FS* a fourth proportional to *CS*, *HS* and *RS*? [3]

Why would a square with side *AH* be equivalent to a rectangle whose sides are *RH* and *HS*? [3]



8 Construct an isosceles triangle, given the angle at the vertex and the altitude to the base. [To receive credit construction lines must be shown.] [12]

9 Find the difference between the areas of a circle and a square, each of whose perimeters is 22 feet. [$\pi = \frac{22}{7}$] [12]

10 For *each* of the following theorems (no proof required) draw the figure and state the hypothesis and conclusion *in terms of letters on the figure:*

a If two lines are tangent to a circle at the extremities of a diameter, and if from the points of contact secants are drawn terminated respectively by the opposite tangent and intersecting the circumference at the same point, the diameter is a mean proportional between the tangents. [6]

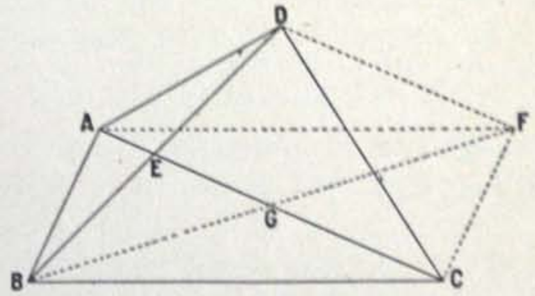
b The sum of the perpendiculars from any point within an equilateral triangle to its sides is equal to the altitude. [6]

11 The sides of a triangle are 8″, 10″ and 12″ respectively. If a line 9″ long parallel to the longest side terminates in the other two sides, find the segments into which it divides them. [12]

12 Assign a reason to *each* of the eight steps given in the following proof:

THEOREM

If one diagonal of a quadrilateral bisects the other diagonal, it bisects the quadrilateral.



Given The quadrilateral ABCD, whose diagonals AC and BD meet in E, making BE = ED.

To prove $\triangle ACD \Bumpeq \triangle ACB$ [$\Bumpeq$ means *equal in area.*]

Proof: Draw AF ∥ BC; CF ∥ BA; let F be their point of intersection; draw DF; draw BF meeting AC in G.

1 ABCF is a parallelogram [2]
2 BG = GF [2]
3 BE = ED [2]
4 EG ∥ DF [2]
5 The perpendicular from D to AC = the perpendicular from F to AC [2]
6 $\triangle ACD \Bumpeq \triangle ACF$ [2]
7 $\triangle ACF \Bumpeq \triangle ACB$ [2]
8 $\triangle ACD \Bumpeq \triangle ACB$ [2]