

High School Department

178TH EXAMINATION

PLANE GEOMETRY

Wednesday, June 17, 1903—9.15 a. m. to 12.15 p. m., only

Answer eight questions but no more, including at least one from each of the three divisions. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.

First division 1 Prove that if two parallel lines are cut by a third straight line, the sum of the two interior angles on the same side of the transversal is equal to two right angles.

2 Prove that through three points not in a straight line one circumference and only one can be drawn.

3 Complete and demonstrate the following: an angle formed by two tangents is measured by . . .

4 Prove that if a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

5 Complete and demonstrate the following: the area of a rectangle is equal to . . .

Second division 6 The legs of a right triangle are 21 inches and 28 inches respectively; find (a) the segments of the hypotenuse made by the bisector of the right angle, (b) the altitude on the hypotenuse.

7 The areas of three concentric circles are in the ratio of 1, 2 and 3; the diameter of the largest circle is 24 inches. Find the diameter of *each* of the other circles.

8 A tangent and a secant are drawn to a circle from an exterior point; the tangent is 14 inches long and the whole secant is 4 times its external segment. Find the length of the secant.

9 Find the area of an equilateral triangle inscribed in a circle whose radius is $2\sqrt{3}$.

10 Find the number of degrees in each of the two arcs of a circle, intercepted by two tangents forming an angle of 75° .

Third division 11 Show how to construct a circle of given radius, tangent to a given circle and to a given line.

12 What is the locus of the vertex of a triangle having a constant base and a constant vertical angle? Give proof.

13 Two circles are tangent at *A*; prove that secants drawn through *A* are cut proportionally by the circumferences.

14 Prove that the bisector of an angle formed by two tangents to a circle passes through the center of the circle.

15 Prove that if one of the legs of an isosceles triangle is produced through the vertex by its own length, the line joining the extremity of the line produced, to the nearer end of the base is perpendicular to the base.