

The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 20, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that if two sides of a triangle are equal, the angles opposite these sides are equal. [10]

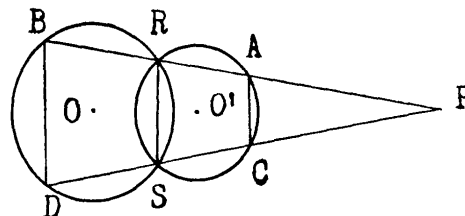
27 Tangents drawn to a circle at points A and B intersect at point P . Prove that the line from P to the center of the circle passes through the mid-point of minor arc AB . [10]

28 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

29 In the accompanying figure circles O and O' intersect at points R and S . Secants from P through R and S intersect the circles at points A and B , and C and D respectively. Chords AC , RS and BD are drawn.

a If the number of degrees in arc RBD is represented by $2x$, express in terms of x the number of degrees in $\angle DSR$, $\angle DBR$, $\angle RSC$ and $\angle CAR$. [4]

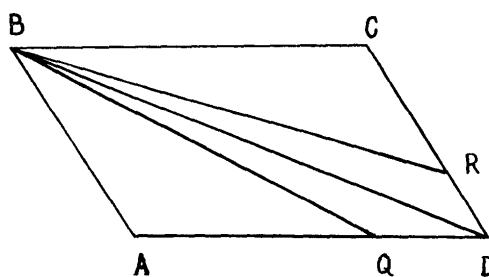
b Prove that $PA : PB = PC : PD$. [6]



30 Given parallelogram $ABCD$ with diagonal BD . Q is a point on AD and R a point on CD such that $AQ : QD = CR : RD = 2 : 1$.

a Prove that triangle ABQ is $\frac{2}{3}$ of triangle ABD . [5]

b Prove that lines BQ and BR divide the parallelogram into three equal parts. [5]



[1]

[OVER]

Part III

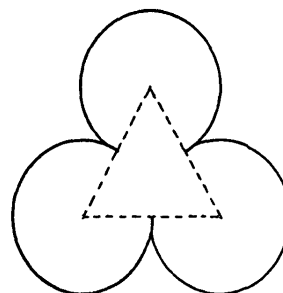
Answer two questions from part III.

- 31 *a* The bases of a trapezoid are 15 and 3 and each of the other sides is 10. Find its altitude. [4]
b A side of a rhombus is 12 and one of its angles is 30° . Find its altitude. [2]
c Show that the trapezoid and the rhombus are equal in area. [4]
- 32 *a* If R represents the radius of the circle circumscribed about a regular pentagon, show that its apothem is equal to $R \cos 36^\circ$ and its perimeter is equal to $10 R \sin 36^\circ$. [2, 2]
b Find to the *nearest integer* the apothem and the perimeter of the pentagon when $R = 5$. [2, 2]
c Using the results found in answer to *b*, find the area of the pentagon. [2]

33 An ornamental window has the form of a trefoil. A trefoil is formed by using the vertices of an equilateral triangle as centers and one half the sides of the triangle as radii and drawing arcs of circles as shown in the figure.

a If the side of the triangle in such a window is 4 feet, find

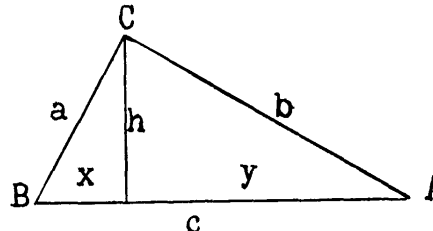
- (1) the area of the triangular part of the window [Answer may be left in radical form.] [2]
 (2) the area of *one* of the sectors in the window [Answer may be left in terms of π .] [4]



b Using $\pi = 3.14$ and $\sqrt{3} = 1.73$, find to the *nearest square foot* the area of the entire window. [4]

34 In the accompanying figure a , b and c represent the sides of triangle ABC ; x and y represent the segments made by the altitude h on side AB ; also $x : h = h : y$. Explain why each of the following statements is true:

- (1) $a^2 = h^2 + x^2$ and $b^2 = h^2 + y^2$ [2]
 (2) $a^2 + b^2 = x^2 + 2h^2 + y^2$ [2]
 (3) $a^2 + b^2 = x^2 + 2xy + y^2$ [2]
 (4) $a^2 + b^2 = c^2$ [2]
 (5) ABC is a right triangle. [2]



[2]

PLANE GEOMETRY

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the diagonal of a square whose side is 5. [Answer may be left in radical form.] 1.....

2 Find the circumference of a circle whose radius is $3\frac{1}{2}$. [Use $\pi = \frac{22}{7}$] 2.....

3 Two angles are complementary and one is 20° larger than the other. Find the number of degrees in the smaller angle. 3.....

4 Angles A and B of parallelogram $ABCD$ are in the ratio 4:5. Find the number of degrees in angle A . 4.....

5 Find the number of degrees in each exterior angle of a regular polygon of 10 sides. 5.....

6 Quadrilateral $ABCD$ is inscribed in a circle and its diagonals intersect at E . If arc $AB = 80^\circ$ and arc $CD = 60^\circ$, find the number of degrees in angle AEB . 6.....

7 In triangle ABC , D is a point on AB , E is a point on AC , and line DE is drawn. If $AB = 8$, $AC = 12$, $DB = 3$ and $EC = 4$, is DE parallel to BC ? [Answer yes or no.] 7.....

8 The altitude on the hypotenuse of a right triangle is 4 and one of the segments of the hypotenuse made by the altitude is 2. Find the other segment of the hypotenuse. 8.....

9 The hypotenuse of a right triangle is 8. Find the median to the hypotenuse. 9.....

10 The areas of two similar triangles are in the ratio 9:25. If an altitude of the smaller triangle is 4, find the corresponding altitude of the larger triangle. 10.....

11 Two chords intersect within a circle. The product of the segments of one chord is 60 and the product of the segments of the other is $15x$. Find x . 11.....

12 In acute triangle DEF , $DE = 10$ and angle $D = 42^\circ$. Find to the nearest tenth the length of the altitude drawn to side DF . 12.....

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Directions (13-17) — Indicate the correct completion for each of the following by writing the letter *a*, *b* or *c* on the dotted line at the right.

- 13 The center of the circle circumscribed about a right triangle lies (a) inside the triangle (b) on one side of the triangle (c) outside the triangle 13.....
- 14 If two tangents are drawn to a circle at the end points of a chord which has an arc of 60° , the triangle formed by the tangents and the chord is (a) acute (b) equilateral (c) obtuse 14.....
- 15 Tangent *PA* and secant *PBC* are drawn to a circle from external point *P*. If chords *AB* and *AC* are drawn, triangle (a) *PAB* is similar to triangle *PAC* (b) *PAC* is similar to triangle *ABC* (c) *PAB* is similar to triangle *ABC* 15.....
- 16 The locus of the centers of circles tangent to a given line at a given point on that line is a (a) point (b) straight line (c) circle 16.....
- 17 A converse of a proposition is (a) always true (b) sometimes true (c) never true 17.....

Directions (18-23) — In the case of each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

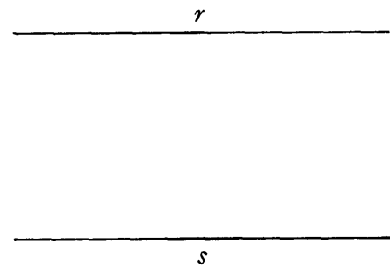
- 18 If the sides of a triangle are unequal, the median drawn to any side is greater than the altitude drawn to that side. 18.....
- 19 The bisectors of two supplementary adjacent angles are perpendicular to each other. 19.....
- 20 The bisector of an angle of a triangle bisects the side opposite this angle. 20.....
- 21 If the diagonals *d* and *d'* of a quadrilateral are perpendicular to each other, the area of the quadrilateral is equal to $\frac{1}{2}dd'$ 21.....
- 22 Two isosceles triangles are similar if they have any angle of one equal to the corresponding angle of the other. 22.....
- 23 If two circles whose centers are *O* and *O'* intersect at points *A* and *B*, chord *AB* is the perpendicular bisector of line segment *OO'*. 23.....

24 The accompanying figure shows the usual method of constructing an angle equal to a given angle. Which two of the following statements are used to prove that angle *A'* is equal to angle *A*?



- (1) Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other. 24.....
- (2) Two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.
- (3) Corresponding parts of congruent triangles are equal. 24.....

25 Given parallel lines *r* and *s*. Construct a line parallel to *r* and *s* and midway between them. [Leave all construction lines on your paper.]



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