

PLANE GEOMETRY

Tuesday, January 27, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

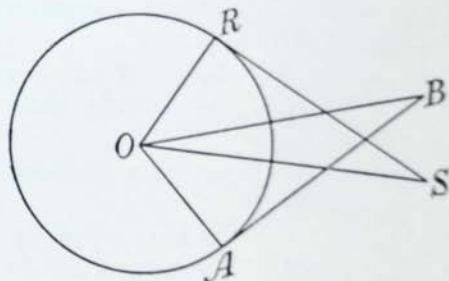
26 Prove that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [10]

27 AB is a diameter of circle O , AC is any chord and radius OC is drawn. Prove that the bisector of angle BOC is parallel to AC . [10]

28 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

29 Given: A and R , points on circle O ,
 $AB \perp$ radius OA , $AB = RS$ and $OB = OS$.
 Prove: RS is tangent to circle O .

[Suggestion: Prove $\triangle ORS$ is a right \triangle .] [10]



Part III

Answer one question from part III.

30 In right triangle ABC , hypotenuse $AB = 12.3$ and angle $B = 40^\circ$

a Find AC to the nearest tenth. [4]

b Find the area of triangle ABC to the nearest integer. [6]

31 In circle O , central angle $AOB = 90^\circ$, arc $AB = 12\pi$ and chord AB is drawn.

a Find radius OA . [3]

b Find the area of triangle AOB . [3]

c Find the area of sector AOB . [Answer may be left in terms of π .] [3]

d Find the area of segment AB . [Answer may be left in terms of π .] [1]

[1]

[OVER]

Part IV

Answer one question from part IV.

32 Each of the five parts of this question is a statement that may be correctly completed by one or more of the given choices. Write the numbers (1) to (5) on your answer paper and after each indicate the correct answer or answers to the corresponding question by writing one or more of the letters a, b, c, d . [10]

[In each of the five parts of the question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

- (1) The diagonals of a parallelogram (a) are equal (b) bisect each other (c) are perpendicular to each other (d) bisect the angles through which they pass
- (2) In a regular polygon (a) the apothem equals the radius of the polygon times the sine of one half an interior angle (b) a central angle equals an exterior angle (c) all the diagonals are equal (d) a circle can be inscribed
- (3) The number of points equidistant from two given parallel lines and at a distance d from point P on one of the lines may be (a) 0 (b) 1 (c) 2 (d) 4
- (4) If two lines are cut by a transversal, the lines will be parallel if (a) any pair of alternate interior angles are equal (b) any two acute angles are equal (c) two interior angles on the same side of the transversal are supplementary (d) two corresponding angles are supplementary
- (5) One and only one triangle can be constructed if the given parts are (a) three angles (b) a side, an adjacent angle and the altitude on the side (c) a side, an adjacent angle and the median to the side (d) two sides and any angle

33 Angle ABC is an acute angle inscribed in circle O . P is a point so located that angle APC is less than angle B . Prove that point P must be outside circle O . [Suggestion: Use the indirect method of proof.] [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 The line segment joining the mid-points of two sides of a triangle is 15. Find the third side of the triangle. 1.....
 - 2 How many degrees are there in an angle formed by a tangent and a secant which intercept arcs of 110° and 42° on a circle? 2.....
 - 3 Find the diagonal of a square whose side is 5. [Answer may be left in radical form.] 3.....
 - 4 The diagonals of a rhombus are 10 and 24. Find a side of the rhombus. 4.....
 - 5 Find the altitude of an equilateral triangle whose side is 8. [Answer may be left in radical form.] 5.....
 - 6 In right triangle ABC , altitude CD is drawn to the hypotenuse AB , making $AD = 3$ and $DB = 12$. Find CD . 6.....
 - 7 The longer side of a rectangle is 10 and a diagonal makes with this side an angle of 35° . Find to the nearest integer the shorter side. 7.....
 - 8 Express the area of an equilateral triangle in terms of its side s . 8.....
 - 9 Express the area of a regular polygon in terms of its apothem a and its perimeter p . 9.....
 - 10 A tangent and a secant are drawn to a circle from the same point. If the entire secant is 16 and its external segment is 4, find the tangent. 10.....
 - 11 In triangle ABC , angle A is obtuse and angle $B = 45^\circ$. Name the shortest side of the triangle. 11.....
 - 12 Write in terms of n the expression for the sum of the interior angles of a polygon of n sides. 12.....
 - 13 Two corresponding sides of two similar triangles are 6 and 24. An altitude of the smaller triangle is 4. Find the corresponding altitude of the larger triangle. 13.....
 - 14 The area of a circle is 49π . Find a side of the inscribed regular hexagon. 14.....
 - 15 The bases of an isosceles trapezoid are 7 and 15. Each leg makes an angle of 45° with the longer base. Find the altitude of the trapezoid. 15.....
- Directions (questions 16-20) — Indicate whether each statement is true or false by writing the word *true* or *false* on the line at the right.
- 16 The exterior angles at the base of an isosceles triangle are obtuse. 16.....
 - 17 Two right triangles are congruent if any two sides of one equal the corresponding sides of the other. 17.....
 - 18 P is the mid-point of arc AB of a circle. Then chord AB is twice chord AP . 18.....
 - 19 If two chords of a circle are perpendicular to each other, one is a diameter. 19.....
 - 20 The mid-point of the hypotenuse of a right triangle is the center of the circle which circumscribes the triangle. 20.....

Directions (questions 21–23) — Indicate the correct answer to each question by writing on the line at the right the letter a , b or c .

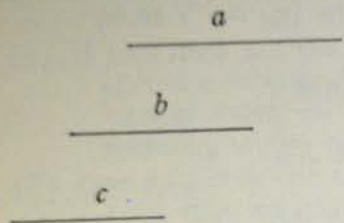
21 A regular polygon is defined as a polygon whose (a) angles are equal
(b) sides are equal (c) sides are equal and whose angles are equal 21.....

22 A triangle and a parallelogram having the same base are equal in area. The altitude of the triangle is (a) one fourth the altitude of the parallelogram
(b) one half the altitude of the parallelogram (c) twice the altitude of the parallelogram 22.....

23 Corresponding sides of two similar triangles are in the ratio 1:9. The ratio of their areas is (a) 1:3 (b) 1:9 (c) 1:81 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 Given line segments a , b and c . Construct line segment x such that $a : b = c : x$



25 Find by construction the locus of points equidistant from points A and B .

A

B