The University of the State of New York
253d HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Wednesday, January 27, 1932 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Fill in the following lines:

Name of school........................................Name of pupil........................................

Detach this sheet and hand it in at the close of the one hour period.

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–18) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1. In proving that the construction for bisecting an angle is correct, the two triangles are proved congruent by one of the three methods, (a) \( s a s \), (b) \( a s a \) or (c) \( s s s \); the method used in this case is \( ... \). [Answer \( a, b \) or \( c \).]

2. If two lines are cut by a transversal so that a pair of alternate interior angles are equal, the interior angles on the same side of the transversal are \( ... \).

3. If the sum of two angles of a triangle equals the third angle, the triangle is a \( ... \) triangle.

4. If, in quadrilateral \( ABCD \), \( AB = 10 \), \( BC = 8 \), \( DA = 8 \) and \( CD = 10 \), then the quadrilateral must be a \( ... \).

5. A diameter bisecting a chord which is not a diameter, is \( ... \) to the chord.

6. An angle formed by two lines which meet a circle but intersect outside the circle is measured by one half the \( ... \) of the intercepted arcs.

7. Three consecutive sides of an inscribed quadrilateral cut off arcs of \( 70^\circ \), \( 90^\circ \) and \( 120^\circ \); the obtuse angle formed by the diagonals of the quadrilateral contains \( ... \) degrees.

8. Two circles have the same center; the locus of the centers of all circles tangent to both is a \( ... \).

9. The lengths of the sides of a triangle are 4, 5 and 6. The perimeter of a similar triangle is 30. The area of the second triangle is \( ... \) times the area of the first.

10. Two chords, \( AB \) and \( CD \), intersect at \( E \) inside a circle. \( AE = 6 \), \( EB = 4 \) and \( CE = 3 \). If a length \( EX = 7 \) is laid off on \( ED \), extended if necessary, the point \( X \) is \( ... \) the circle. [Answer \( \text{inside, on or outside} \).]

11. If the altitude \( CD \) is drawn to the hypotenuse \( AB \) of the right triangle \( ABC \), the last term of the proportion beginning \( AB:AC = AC: \) is \( ... \).

12. A baseball diamond is a square 90 feet on a side. The distance from home plate to second base is \( ... \) feet. [Find answer correct to the nearest foot.]

[3]
Plane Geometry — concluded

13 When the angle of elevation of the sun is 58°, the height of a pole that casts a shadow 10 feet long is ... feet.

14 If the base of a triangle is doubled while the area remains unchanged, the altitude is multiplied by ....

15 The altitude of a parallelogram is \( a \) and its base is \( b \). The side of an equal square is \( s \). The proportion expressing the relation between \( a, b \) and \( s \) is ....

16 A rectangle whose base is 16 is inscribed in a circle whose radius is 10; the altitude of the rectangle is ....

17 A belt, which does not cross, goes around two equal wheels whose radii are 4 inches each and whose centers are 9 inches apart. The length of the belt is ... inches. [The answer may be left in terms of \( \pi \].]

18 If in quadrilateral \( ABCD, AB = BC \), and angle \( A \) is larger than angle \( C, AD \) is ... than \( CD \). [Suggestion — Draw \( AC \].]

Directions (questions 19–20) — Leave all construction lines on the paper.

19 Divide the line \( CD \) into two segments proportional to lines \( a \) and \( b \).

\[
\begin{align*}
\text{a} & \\
\text{b} & \\
\end{align*}
\]

20 In triangle \( ABC \), construct the median to side \( BC \).
PART II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that, if two angles of a triangle are equal, the sides opposite these angles are equal. [12]

22 Prove that tangents drawn to a circle from an external point are equal. [12]

23 On a given line segment as a side, construct a quadrilateral similar to a given quadrilateral. [Show all construction lines.] [12]

24 Prove that the apothem of an inscribed, regular hexagon equals half the side of an equilateral triangle formed by joining alternate vertices of the hexagon. [12]

25 Prove that if two circles are tangent externally, they cut off proportional lengths from any two secants that pass through their point of contact. [12]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of π and radicals unless otherwise stated.

26 Find the cost, at 20 cents a square foot, of paving a circular track 10 feet wide, if the distance around the track on its inner edge is 628 feet. [Use π = 3.14] [12]

27 The cross section of a dam is a trapezoid whose longer base is 20 feet. The nonparallel sides are each 12 feet, and with this base they form angles of 60° each. Find the area of the trapezoid. [12]

28 An airplane that can fly 100 miles an hour in still air, is headed due east in a wind which is blowing due north 30 miles an hour. Find the distance the airplane will travel in three hours, and the angle, to the nearest degree, which its path makes with due east. [8, 4]

29 In the right triangle ABC, CH and CM are respectively the altitude and the median to the hypotenuse AB, and angle B is 32°.

a Find the number of degrees in angle HCM. [8]

b To what number of degrees must angle B be changed in order that altitude CH should coincide with median CM? [4]