The University of the State of New York

247th High School Examination

PLANE GEOMETRY

Wednesday, January 22, 1930 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Name of school........................................Name of pupil........................................

Detach this sheet and hand it in at the close of the one hour period.

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The bisectors of two complementary adjacent angles form an angle of . . . degrees.

2 If the area of an equilateral triangle is $25\sqrt{3}$, the length of one side is . . .

3 If two isosceles triangles have a common base, the line determined by their vertices is . . . to the base.

4 The perimeter of a regular hexagon inscribed in a circle is 42 inches; the circumference of the circle in terms of $\pi$ is . . . inches.

5 In the rhombus $ABCD$, if angle $B = 120^\circ$ and diagonal $BD = 10$, then the length of one side of the rhombus is . . .

6 $AB$ is a diameter and $AK$ a chord in a circle such that angle $BAK = 55^\circ$; the number of degrees in arc $AK$ is . . .

7 In an isosceles triangle, each of the two equal sides is 12 and the angle included by them is $120^\circ$; the length of the base is . . .

8 The bases of an isosceles trapezoid are 20 and 30 and the angles at the extremities of the longer base are each $45^\circ$; the altitude is . . .

9 The tangent to a circle at a vertex of an inscribed regular pentagon makes an acute angle of . . . degrees with one side.

10 If a tangent and a secant drawn from the same point to a circle are 6 inches and 18 inches long respectively, the length of the external segment of the secant is . . . inches.

11 If any angle of one isosceles triangle equals the corresponding angle of another isosceles triangle, then the two triangles are . . .

12 The equilateral quadrilateral that is not a regular polygon is called a . . .

13 Two altitudes of a triangle fall outside the triangle if the triangle is . . .

14 The corresponding bases of two similar triangles are 2 and 3; if the area of the first triangle is 12, the area of the second is . . .
15 In triangle $ABC$, $AB$ is greater than $AC$, and the bisector of angle $A$ meets $BC$ in $D$; then angle $BDA$ is . . . than angle $CDA$.

16 If a diagonal is drawn in a quadrilateral whose opposite sides are equal, which of the following reasons would be used in proving that the triangles formed are congruent?  
(a) $s a s$; (b) $a s a$; (c) $s s s$  

17 Which two of the following sets of numbers taken as sides of triangles will form similar triangles?  
(a) $8, 15, 27$; (b) $4, 7, 9$; (c) $4, 7\frac{1}{2}, 13\frac{1}{2}$

Directions (questions 18–20) — Leave all construction lines on the paper.

18 Construct the mean proportional between lines $a$ and $b$.

19 Given angle $BCA = 90^\circ$; construct an angle of $75^\circ$ with vertex at $C$ and with $CB$ as one of its sides.

20 Construct the locus of the centers of circles tangent to line $m$ at point $P$. 
PLANE GEOMETRY

Wednesday, January 22, 1930

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.
The minimum time requirement is five recitations a week for a school year.
Name the author of the textbook you have used in plane geometry.

PART II

Answer five questions from part II, including three questions from group I and two questions from Group II.

Group I

Answer three questions from this group.

21 Prove that if two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other. [12]

22 In triangle ABC, R and S are the mid-points of sides AC and BC respectively. Line RS is extended its own length through S to point P and line PB is drawn.
Prove (a) BP = AR, (b) BP is parallel to AR. [8, 4]

23 a Given two points, A and B, on an indefinite line m and two other lines whose lengths are represented by a and b, with b greater than a. By actual construction locate a point C that shall be a distance a from line m and a distance b from the mid-point of segment AB. [10]

b How many such points are there? [2]

24 In triangle ABC, side AB is greater than side AC. If the bisectors of angles B and C meet in point P, prove that PB is greater than PC. [12]

25 D is any point in side AC of triangle ABC and line DE joins D to any point E on the extension of side BA through A.
Prove \[
\frac{\text{triangle } ADE}{\text{triangle } ABC} = \frac{AD \times AE}{AC \times AB} \] [12]
[Suggestion: Draw the altitudes of triangles ADE and ABC from vertices D and C respectively.]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of \pi and radicals unless otherwise stated.

26 ABCD is a parallelogram with side AB = 18, side AD = 12 and angle A = 60°.

a Find the area of parallelogram ABCD. [10]

b If M, any point in CD, is joined to A and B, find the area of triangle ABM. [2]

27 ABCD is a rectangle inscribed in a circle, the minor arc of chord AB containing 98°. Diagonal CA extended through A meets in point E the tangent drawn at D. Find the number of degrees in angle E. [12]

28 Two circles have radii of 6 inches and 8 inches. The circumference of a third circle is equal to the combined circumferences of the two circles. What is the area of this third circle? [Use \pi = 34] [12]

29 A boat travels north 20 miles, then east 8.1 miles and then north 16 miles; how far from the starting point is the final position of the boat? [12]