PLANE GEOMETRY

Tuesday, January 19, 1915 — 9.15 a.m. to 12.15 p.m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in your study of plane geometry.

Answer eight questions, including question 13.

1. Prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles. \([12]\)]

2. Prove that the diameter perpendicular to a chord bisects the chord and also its subtended arc. \([12]\)]

3. Prove that two similar polygons may be divided into the same number of triangles similar each to each and similarly placed. \([12]\)]

4. Prove that in any right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. \([12]\)]

5. For each of the following propositions, draw the figure and state the hypothesis and the conclusion in terms of the letters on the figure:

   a. The internal common tangents of two non-intersecting circles meet on their line of centers. \([6]\)]

   b. The square of the bisector of an angle of a triangle is equal to the product of the sides of this angle diminished by the product of the segments of the third side made by the bisector. \([6]\)]

6. A circle is inscribed in an isosceles trapezoid whose bases are 6 and 18. Find the area of the portion of the trapezoid not included in the circle. [First find the side of the trapezoid.] \([12]\)]

7. Two sides of a triangle are 8 and 12 and the altitude on the third side is 6. A similar triangle has the side homologous to 8 equal to 10. Compute the other two sides and the altitude to the longest side in the second triangle. \([13]\)]

8. Prove that if the bisectors of the base angles of an isosceles triangle \(ABC\) meet the opposite sides in \(E\) and \(F\), \(EF\) is parallel to the base of the triangle. \([12]\)]

9. In an isosceles triangle construct the two circles as shown in the accompanying figure. [All construction lines must be shown.] \([12]\)]
10. If any point on the base $BC$ of the triangle $HBC$. $P$ is the mid-point of $BC$. $PD$ is drawn parallel to $HF$. Prove that the triangle $DFC$ equals one half the triangle $HBC$. [12]

11. a. If an isosceles triangle is obtuse, which is its longest side? [2]

b. How many diagonals can be drawn from one vertex of a polygon of $n$ sides? [2]

c. Under what conditions will the circles having radii $a$ and $b$ respectively, intersect in two distinct points [2] and when will they be externally tangent [2]?

d. Define similar polygons. [2]

e. Write two theorems that are included in the following statement: If through a point $P$ a line is drawn cutting a circle in $A$ and $B$, then the product $PA \times PB$ is the same for all such lines. [2]

12. Two equal chords are produced till they meet. The angle formed by extending the chords is $180^\circ$ and the smaller arc intercepted by them is $15^\circ$ of the circumference. Find each angle of the quadrilateral formed by joining the ends of the chords. [12]

13. Assign a reason to each of the eight steps given in the following proof:

**THEOREM**

*If in a quadrilateral the diagonals are equal and two sides are parallel, the other sides are equal.*

Given: The quadrilateral $CLRS$ whose diagonals $CR$ and $SL$ are equal and two sides $CL$ and $SR$ parallel.

To prove: $CS = LR$

**Proof:**

1. $CLS$ is a parallelogram [2]
2. $NC = CL$ and $CN = LS$ [2]
3. $CN = CR$ [2]
4. $\angle CNR = \angle CRN$ [2]
5. $\angle RCL = \angle CRN$ [2]
6. $\angle CNR = \angle RCL$ [2]
7. $\triangle CSN = \triangle CLR$ [2]
8. $CS = LR$ [2]