

PLANE GEOMETRY

Tuesday, January 20, 1914—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry. Name the author of the textbook you have used in your study of plane geometry.

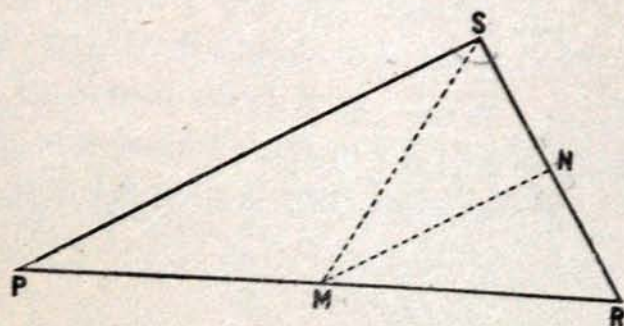
Answer the first question and seven of the others.

Assign 16 credits to the first question and 12 credits to each of the others.

1 Assign a reason to *each* of the eight steps given in the following proof:

THEOREM

If one acute angle of a right triangle is double the other, the hypotenuse is double the shorter side.



Given rt. $\triangle RSP$ with $\angle R = 2\angle P$

To prove: $RP = 2RS$

Proof:

Let M be the mid point of RP. Draw MS. Draw $MN \parallel PS$.

1 $RN = NS$

2 $MN \perp RS$

3 $MR = MS$

4 $\angle R + \angle P = 90^\circ$

5 $3\angle P = 90^\circ$

6 $\angle R = 60^\circ$

7 $RM = RS$

8 $RP = 2RS$

2 Prove that if two sides of a triangle are unequal, the angles opposite are unequal and the greater angle is opposite the greater side.

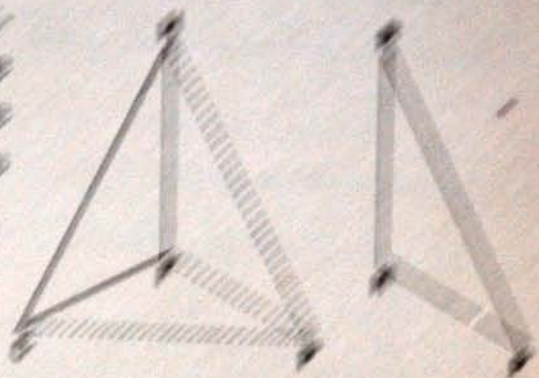
3 Prove that if a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

6 Prove that the bisector of the exterior angle of a triangle divides the opposite side externally into segments which are proportional to the other two sides.

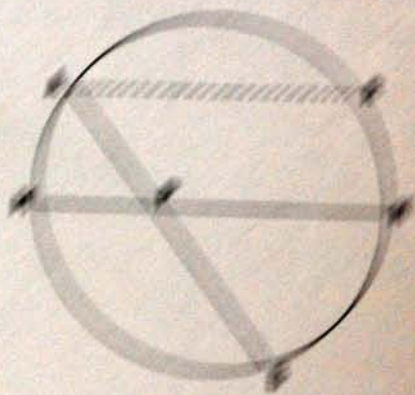
7 Construct a common external tangent to two unequal circles. (Show all construction lines. No credit will be granted for explanation.)

8 From the accompanying figs, prove that two triangles are equal if three sides of the one are equal respectively to three sides of the other.

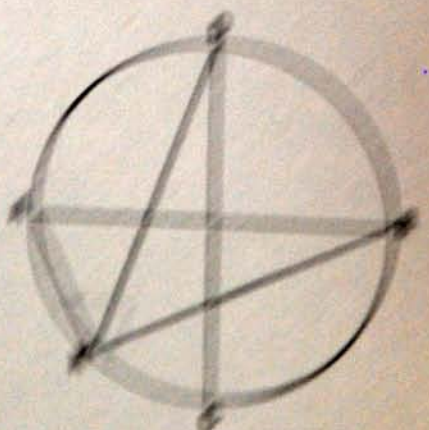
$$\begin{aligned} \text{Given } AC &= XZ \\ AB &= XZ \\ CB &= XY \end{aligned}$$



9 From the accompanying figure, prove that an angle formed by two chords which intersect within a circle is measured by one half the sum of the intercepted arcs. (Assume that AB is parallel to CD .)



10 In the accompanying figure, AB and CD are perpendicular diameters of a circle. E is any point on the arc AC . Prove that AE bisects $\angle AEB$.



11 The sides of a triangle are b, c, d ; and the radius of the circle whose area is equal to the area of the triangle.

12 Two equal circles (radius r) intersect so that their common chord is equal to the radius; find the area of the figure that lies within both circles.

13 Prove that if a straight line bisects a chord and its subtended arc, it is perpendicular to the chord.