NINTH YEAR MATHEMATICS

37. On your answer paper write the letters a through e and next to each letter write the number of the property of the real number system, chosen from the list below, which justifies each of the statements a through e below: [10]

Properties

(1) additive identity property
(2) multiplicative inverse property
(3) distributive property of multiplication over addition
(4) associative property of addition
(5) associative property of multiplication
(6) commutative property of addition
(7) commutative property of multiplication

\[
\begin{align*}
a & : 2 \left(\frac{1}{2}\right) = 1 \\
b & : 2(1 + 3) = 2(1) + 2(3) \\
c & : 2 + 1 = 1 + 2 \\
d & : (1 + 2) + 3 = 1 + (2 + 3) \\
e & : 2 + 0 = 2 \\
\end{align*}
\]

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Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided.

1. If \(a = 2\) and \(b = 3\), find the value of \(2a + b\).  

2. If four more than twice a number is 32, find the number.  

3. The sum of two numbers is 23. One number is represented by \(x\). Represent the other number in terms of \(x\).  

4. Solve for \(t\): \(8t = 0\)  

5. Find the solution set: \(4(x - 2) = 12 - x\)  

6. Solve for \(d\): \(\frac{d - 4}{d} = \frac{5}{6}\)  

7. Solve the following system of equations for \(x\):
\[
\begin{align*}
x - y & = 3 \\
x + y & = 1 \\
\end{align*}
\]

8. Factor completely: \(3ax + 6a\)  

9. One gallon of paint is needed to cover 350 square feet of surface. How many gallons of paint will be needed to cover 1,050 square feet?
10. Express the product \((2x + 1) (x - 3)\) as a trinomial.

11. If the replacement set for \(x\) is \(\{7, 8, 9, 10\}\), find the solution set of \(x - 1 < 8\).

\[8\]

12. Find the value of \(\frac{8}{3 - x}\) if \(x = -1\).

13. Express the sum of \(4\sqrt{3}\) and \(\sqrt{75}\) as a single term in radical form.

14. Express as a single fraction: \(\frac{6}{7} \div \frac{x}{y}\)

15. If \(x = -4\), find the value of the expression \(x + \left|\text{x}\right|\).

16. How many square feet are there in the area of a square if its perimeter is 20 feet?

17. Express the fraction \(\frac{3x - 9}{x^2 - x - 6}\) in lowest terms.

18. The hypotenuse of a right triangle is 13 feet and one leg is 12 feet. Find the number of feet in the length of the other leg.

19. What is the slope of the graph of \(v = -3x + 4\)?

Directions (20-30): Write the numeral preceding the expression that best completes each statement or answers each question.

20. The reciprocal of \(\frac{5}{x - 2}\) is (1) \(-5\) (2) \(\frac{5}{2 - x}\)

(3) \(\frac{x - 2}{5}\) (4) \(\frac{5}{x - 2}\)

21. Given the set \(S = \{0, 2, 4, 6, 8\}\), which is not a subset of \(S\)?

(1) \(\{2, 6, 8\}\) (2) \(\{6, 7, 8\}\) (3) \(\{0\}\) (4) \(\{\}\)

22. The product of \(4x^2y^3\) and \(2x^3y^4\) is (1) \(8x^5y^7\) (2) \(8x^6y^7\)

(3) \(16x^5y^7\) (4) \(6x^5y^7\)

23. If \(5r - s\) is subtracted from \(6r - 3s\), the result is (1) \(r - 4s\)

(2) \(-r + 2s\) (3) \(r - 2s\) (4) \(11r - 4s\)

24. Which is equal to 2? (1) \(\frac{|-6|}{-3}\) (2) \(-\frac{6}{-3}\)

(3) \(-\frac{6}{3}\) (4) \(\frac{3}{0}\)

25. The graph of \(x + y = 4\) crosses the y-axis at the point whose coordinates are (1) \((0, -4)\) (2) \((-4, 0)\) (3) \((0, 4)\) (4) \((4, 0)\)

26. An equation which has the solution set \(\{2, 3\}\) is

(1) \((x - 2) (x - 3) = 0\) (2) \((x + 2) (x + 3) = 0\)

(3) \(x(2x - 3) = 0\) (4) \(2(x - 3) = 0\)
27. If \( A = \frac{bh}{2} \), then \( h \) can be expressed in terms of \( A \) and \( b \) as

- \( \frac{2b}{A} \)
- \( \frac{Ab}{2} \)
- \( 2Ab \)
- \( \frac{2A}{b} \)

28. In the accompanying diagram, the legs of right triangle \( ABC \) are 8 and 6, and the hypotenuse is 10. Cosine \( A \) equals

- \( 4/5 \)
- \( 5/4 \)
- \( 4/3 \)
- \( 3/4 \)

29. The set of positive even integers, \( \{2, 4, 6, 8, 10, \ldots\} \), is closed under

- \( 1 \) addition and subtraction
- \( 2 \) multiplication and division
- \( 3 \) addition and multiplication
- \( 4 \) subtraction and division

30. The figure shows a rectangle with a square cut from one corner.

Expressed in terms of \( x \) and \( y \), the area of the shaded region is

- \( xy - \frac{1}{2}y^2 \)
- \( xy - y^2 \)
- \( x^2 - \frac{1}{3}xy \)
- \( y^2 - \frac{1}{3}xy \)

**Part II**

Answer four questions from this part. Show all work unless otherwise directed.

31. Answer either \( a \) or \( b \) but not both:

\( a \) Using a set of coordinate axes, graph the solution set of the following system of inequalities and label the solution set \( A \):

\[
\begin{align*}
y &< 3x - 6 \\
y &< -2x + 4
\end{align*}
\]

\( OR \)

\[
\begin{align*}
y &= 2x + 4 \\
y &= -x + 1
\end{align*}
\]

\( b \) Solve graphically and check:

\[
\begin{align*}
y &= 2x + 4 \\
y &= -x + 1
\end{align*}
\]

32. Answer both \( a \) and \( b \):

\( a \) From the sum of \( 2x^2 + 5x - 3 \) and \( 2x + 1 \) subtract the sum of \( -x^2 + 6x - 2 \) and \( 2x^2 + 5 \). Express your result as a trinomial.

\( b \) Solve for \( n \) and check:

\[
\frac{n + 3}{2} + \frac{n - 2}{3} = 5
\]

33. The length of a rectangle is 2 feet longer than its width. If the area of the rectangle is 48 square feet, find the number of feet in the width. \( Only an algebraic solution will be accepted. \)
34. Write an equation or a system of equations which can be used to solve each of the following problems. In each case state what the variable or variables represent. [Solution of the equations is not required.]

a Mr. A and Mr. B leave at the same time from points that are 300 miles apart and travel toward each other. If Mr. A travels at 60 miles per hour and Mr. B at 40 miles per hour, in how many hours will they meet? [5]

b Part of $5,000 is invested at 6% and the remainder at 5%. The total annual income from both investments is $285. Find the number of dollars invested at each rate. [5]

35. Answer both a and b:

a When a 36-foot pole casts a 75-foot shadow, what is the measure of the angle of elevation of the sun to the nearest degree? [5]

b A 20-foot ladder makes an angle of 76° with the level ground, as shown in the diagram. To the nearest foot, what is the distance, \( x \), from the top of the ladder to the ground? [5]

36. One month a school store sold 15 pennants and 10 shirts for a total of $60. The next month it sold 25 pennants and 20 shirts for a total of $110. What was the selling price, in dollars, of one pennant? [Only an algebraic solution will be accepted.] [5, 5]

37. The replacement set for \( x \) for each of the open sentences listed below is \{-3, -2, -1, 0, 1, 2, 3\}. On your answer paper, write the letters \( a \) through \( e \), and next to each write the solution set of each open sentence. [Each answer must be a subset of the replacement set.] [10]

(a) \(|x| = 3\)
(b) \(x^2 - 1 = 0\)
(c) \(x + 3 < 2\)
(d) \(3x - 1 = 0\)
(e) \(x^2 - x - 6 = 0\)