NINTH YEAR MATHEMATICS

June 15, 1984

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the answer sheet.

1. If three times a certain number is increased by 4, the result is 19. What is the number? 1

2. Solve for $x$: $2x + 7 = 5x - 14$ 2

3. Factor: $x^2 - 5x - 24$ 3

4. If $\cos A = .8576$, find the measure of angle $A$ to the nearest degree. 4

5. Solve for $x$: \[ \frac{x}{2} - \frac{x}{3} = \frac{1}{6} \] 5

6. Find the numerical value of $| -3 | + | 7 | - | -7 |$. 6

7. If $S = 4t - 2y$, find $S$ when $t = 2$ and $y = -1$. 7

8. Factor: $49 - x^2$ 8

9. What is the $y$-intercept of the line whose equation is $y = 3x - 5$? 9

10. Solve for $x$: $0.3x - 6 = 0.9$ 10

11. A vertical pole casts a shadow 10 feet long. At the same time, a man 6 feet tall casts a shadow 4 feet long. What is the height, in feet, of the pole? 11

12. The solution set of the equation $(y - 3)(y - b) = 0$ is (3,5). What is the value of $b$? 12

13. Divide $12x^3 - 24x^2 + 3x$ by $3x$. 13

14. Find $\sqrt{53}$ to the nearest tenth. 14

15. Solve the following system of equations for $y$:
   \[
   \begin{align*}
   3y + x &= 13 \\
   y + x &= 1
   \end{align*}
   \] 15

16. Solve for $x$ in terms of $a$, $b$, and $c$: \[ \frac{cx}{a} = b \] 16

17. The sum of the measures of two angles is $90^\circ$. If the measures of the two angles are in the ratio 1:5, find the measure, in degrees, of the smaller angle. 17

18. Subtract $x^2 - 6x + 5$ from $2x^2 - 3x + 2$. 18
Directions (19-30): Write in the spaces provided on the answer sheet the numeral preceding the expression that best completes each statement or answers each question.

19. Which is a rational number?
   (1) \( \sqrt{8} \)  (2) \( \sqrt{12} \)
   (3) \( \sqrt{16} \)  (4) \( 20 \)
   19.____

20. The product of \( 3x^3 \) and \( 2x^7 \) is
   (1) \( 5x^{12} \)  (2) \( 6x^{12} \)
   (3) \( 5x^{35} \)  (4) \( 6x^{35} \)
   20.____

21. If \( p \) and \( q \) represent natural numbers, which expression must represent another natural number?
   (1) \( p + q \)  (2) \( \frac{p}{q} \)
   (3) \( p - q \)  (4) \( \frac{q}{p} \)
   21.____

22. The product of \( (2 + a) \) and \( (3 - b) \) is equal to
   (1) \( 6 + ab - b^2 \)  (2) \( 5 + ab + 3a - 2b \)
   (3) \( 6 - 2b + 3a - ab \)  (4) \( 6 - ab \)
   22.____

23. The multiplicative inverse of \( \frac{1}{6} \) is
   (1) 1  (2) \( \frac{1}{6} \)  (3) -8  (4) 8
   23.____

24. The base of an isosceles triangle is 4 more than one of its legs.
   If \( x \) represents the length of one of its legs, then the perimeter of the triangle is represented by
   (1) \( 4 - x \)  (2) \( 3x + 4 \)  (3) \( 3x - 4 \)
   (4) \( 3x + 8 \)
   24.____

25. For which value of \( x \) is the fraction \( \frac{x + 3}{x - 5} \) undefined?
   (1) -3  (2) -5  (3) 3  (4) 5
   25.____

26. Which equation illustrates the associative property of addition?
   (1) \( 3 + 5 = 5 + 3 \)  (2) \( 2 + (-2) = 0 \)
   (3) \( 2(3 + 4) = 2(3) + 2(4) \)  (4) \( (3 + 4) + 7 = 3 + (4 + 7) \)
   26.____

27. The sum of \( 3\sqrt{3} \) and \( \sqrt{12} \) is
   (1) \( 5\sqrt{3} \)  (2) \( 7\sqrt{3} \)
   (3) \( 3\sqrt{15} \)  (4) \( \sqrt{21} \)
   27.____

28. The expression \( (-3x^2y^3)^3 \) is equivalent to
   (1) \( -3x^6y^6 \)  (2) \( -9x^6y^9 \)
   (3) \( -27x^6y^6 \)  (4) \( -27x^6y^9 \)
   28.____

29. Which coordinates represent a point on the graph of \( y = 4x - 14 \)?
   (1) \( (-2,6) \)  (2) \( 2,-6 \)  (3) \( (-6,2) \)  (4) \( (6,-2) \)
   29.____

30. Which inequality is represented by the graph below?

\[ -4 -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

(1) \( -2 \leq x \leq 6 \)  (2) \( -2 \leq x < 6 \)
(3) \( -2 < x < 6 \)  (4) \( -2 \leq x < 6 \)
   30.____
Answer four questions from this part.
Show all work unless otherwise directed.

31. On the same set of coordinate axes, graph the following system of inequalities and label the solution set S.
\[
\begin{align*}
2x + y & \leq 6 \\
y & \geq 2
\end{align*}
\] [8, 2]

32. Answer both a and b.
   a Solve for \( x \) and check:
   \[
   \frac{2}{x} - \frac{x - 1}{3x} = 2
   \] [5, 1]

   b Perform the indicated operation and express the result in simplest terms:
   \[
   \frac{x + 3}{2x + 4}, \quad \frac{x^2 - x - 6}{2x + 6}
   \] [4]

33. Write an equation or a system of equations that can be used to solve each of the following problems. In each case, state what the variable or variables represent. [Solution of the equations is not required.]
   a Beth paid $3.30 for stamps. Some of the stamps cost 20 cents each and the rest cost 13 cents each. If the number of 20-cent stamps was 10 less than twice the number of 13-cent stamps, how many of each kind did she buy? [5]

   b A cargo plane leaves an airport and travels east at 180 miles per hour. Two hours later a jet plane leaves the same airport and travels in the same direction at 900 miles per hour. How long will it take the jet plane to catch up to the cargo plane? [5]

34. The sum of the squares of two positive consecutive odd integers is 130. Find the integers. [Only an algebraic solution will be accepted.] [5, 5]

35. The denominator of a fraction is 3 times the numerator. If 8 is added to the numerator and 6 is subtracted from the denominator, the value of the resulting fraction is \( \frac{8}{9} \). Find the original fraction [Only an algebraic solution will be accepted.] [6, 4]

36. Answer both a and b.
   a Find to the nearest foot the height of a building that casts a shadow of 80 feet when the angle of elevation of the Sun is 42°. [5]

   b In right triangle \( \triangle ABC \), angle \( C \) is a right angle. The ratio of leg \( AC \) to hypotenuse \( AB \) is 4 to 5. Find the measure of angle \( B \) to the nearest degree. [5]
37. The replacement set for $x$ for each of the open sentences below is $\{-2,-1,0,1,2\}$. On your answer paper, write the letters $a$ through $e$, and next to each letter write the solution set of the open sentence. [Each answer must be a subset of the replacement set.] [10]

\[
\begin{align*}
  a & \quad (x - 1)(x + 2) = 0 \\
  b & \quad \frac{4}{|x|} = 2 \\
  c & \quad 3x - 3 > x + 1 \\
  d & \quad x^2 = 1 \\
  e & \quad -2x \geq 4
\end{align*}
\]