NINTH YEAR MATHEMATICS

January 24, 1983

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the answer sheet.

1. Solve for \( x \): \( 3x + 5 = 5x - 3 \)

2. What is the multiplicative inverse of \(-4\)?

3. Factor: \( x^2 - 8x + 12 \)

4. Solve for \( x \) in terms of \( a \) and \( b \): \( 2x - b = a \)

5. Find the positive root of \( 3x^2 = 27 \).

6. Find the value of \( \sqrt{41} \) to the nearest tenth.

7. Solve the following system of equations for \( x \):
   \[
   \begin{align*}
   x + 2y + 1 &= 0 \\
   x - 2y + 3 &= 0
   \end{align*}
   \]

8. Solve for \( d \): \( \frac{d}{2} + 6 = 4 \)

9. Solve for \( x \): \( 0.07x + 0.02 = 1 \)

10. If the replacement set is \( \{1, 3, 5, 7\} \), find the solution set of the inequality \( 2x + 4 > 10 \).

11. What is the height, in feet, of a tree which casts a shadow 40 feet long at the same time that a 3-foot vertical stake casts a shadow 5 feet long?

12. The distance between Kingston and Buffalo is 320 miles. If Paul has traveled \( 5/8 \) of that distance, how many miles has he traveled?

13. Multiply and express as a trinomial: \( (2y - 5)(y - 4) \)

14. Express in simplest terms: \( \frac{2x + 6}{8} \cdot \frac{2}{x + 3} \)

15. Subtract \( 16x - y \) from \( 18x + y \).

16. If $6,300 is invested at an annual interest rate of 7%, what is the amount of interest earned in one year?

17. The point whose coordinates are \( (3, k) \) lies on the graph of \( 2x + y = 5 \). What is the value of \( k \)?

18. Express as a single fraction in simplest form: \( \frac{2}{3x} + \frac{4}{x} \)

19. Find the slope of the graph of the equation \( y - 4x = 8 \).
Directions (20-30): Write in the space provided on the answer sheet the numeral preceding the expression that best completes each statement or answers each question.

20. Which is the greatest common factor of 16, 20, and 44? (1) 880 (2) 2 (3) 44 (4) 4

21. If \( y = \frac{1}{2} \), the value of \((8y)^2\) is (1) 8 (2) 2 (3) 16 (4) 4

22. Which is a subset of the set of natural numbers? (1) \( \{1, \frac{1}{2}, \frac{1}{3}, \ldots\} \) (2) \( \{2, 4, 6, \ldots\} \) (3) \( \{-1, -2, -3, \ldots\} \) (4) \( \{1, \sqrt{2}, \sqrt{3}, \ldots\} \)

23. In the accompanying graph, which point has coordinates (5, -4)?

(1) A (2) B (3) C (4) D

24. The value of \(|-9| - 5|\) is (1) 14 (2) -14 (3) -4 (4) 4

25. The width of a rectangle is represented by \( x \) and the length by 4\( x \). The area of the rectangle is (1) 10\( x \) (2) 2\( x^2 \) (3) 5\( x \) (4) 4\( x^2 \)

26. If \( n - 2 \) represents an even integer, the next larger even integer is (1) \( n \) (2) 2\( n - 4 \) (3) \( n - 1 \) (4) \( n - 4 \)

27. The legs of a right triangle are 4 and 8. The length of the hypotenuse is (1) \( \sqrt{80} \) (2) \( \sqrt{48} \) (3) 32 (4) 12

28. In right triangle ABC, angle C = 90°, AB = 13, AC = 5, and BC = 12. The value of \( \cos A \) is (1) 5/12 (2) 5/13 (3) 12/13 (4) 13/5

29. If the diameter of a circle is 15, then the circumference of the circle is (1) 7.5\( \pi \) (2) 15\( \pi \) (3) 30\( \pi \) (4) 225\( \pi \)

30. The number of days in 2\( x \) weeks is (1) \( \frac{2x}{7} \) (2) 7\( x \) (3) 14 (4) 14\( x \)
Answer four questions from this part. Show all work unless otherwise directed.

31. On the same set of coordinate axes, graph the following system of inequalities and label the solution set $A$.
\[\begin{align*}
2y & \geq x - 6 \\
x + y & < 2
\end{align*}\] [8, 2]

32. Answer both $a$ and $b$.
\[\begin{align*}
a & \text{ Solve for } x: \quad x^2 - 5x = 6 \quad [5] \\
b & \text{ Express as a single fraction in simplest form: } \quad \frac{x + 2}{4} - \frac{x - 3}{3} \quad [5]
\end{align*}\]

33. Write an equation or a system of equations that can be used to solve each of the following problems. In each case, state what the variable or variables represent. \textit{[Solution of the equations is not required.]} 
\[\begin{align*}
a & \text{ A car leaves town } A \text{ at } 1:00 \text{ p.m. traveling at } 45 \text{ miles per hour. A second car leaves town } A \text{ at } 2:00 \text{ p.m. traveling the same route at } 55 \text{ miles per hour. How long will it take the second car to overtake the first car?} \quad [5] \\
b & \text{ The sum of the digits of a two-digit number is } 8. \text{ The tens digit is one less than twice the units digit. Find the number.} \quad [5]
\end{align*}\]

34. A man invests $6,000 for one year, part in a savings certificate which pays 16\%, and the remainder in tax-free bonds which pay 9\%. If the annual income from both investments is the same, find the amount invested at each rate. \textit{[Only an algebraic solution will be accepted.]} [5, 5]

35. The junior class at King High School raised $800 from the sale of tickets to a dance. Tickets sold for $1.50 in advance and $2.00 at the door. If 475 tickets were sold, how many of these were sold at the door? \textit{[Only an algebraic solution will be accepted.]} [5, 5]

36. In the accompanying figure, triangle $ABC$ is a right triangle with angle $C = 90^\circ$, angle $A = 37^\circ$, and $AB = 25$.
\[\begin{align*}
a & \text{ Find the length of } AC \text{ to the nearest integer.} \quad [5] \\
b & \text{ Find the length of } BC \text{ to the nearest integer.} \quad [5]
\end{align*}\]

37. The replacement set for $x$ for each of the following open sentences is \{-4, -2, 0, 2, 4\}. On your answer paper, write the letters $a$ through $e$ and next to each letter write the solution set of each open sentence. \textit{[Each answer must be a subset of the replacement set.]} [10]
\[\begin{align*}
a & \quad x^2 = 4 \\
b & \quad |x| = 4 \\
c & \quad 2x - 2 > 1 \\
d & \quad 5x = 2x \\
e & \quad x + 4 = 7x - 2
\end{align*}\]