The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 14, 2007 — 1:15 to 4:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HS For Civil Rights

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

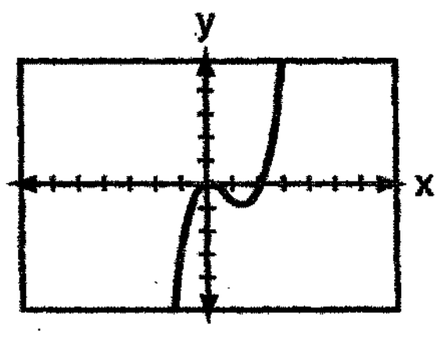
MATHEMATICS B

Part I

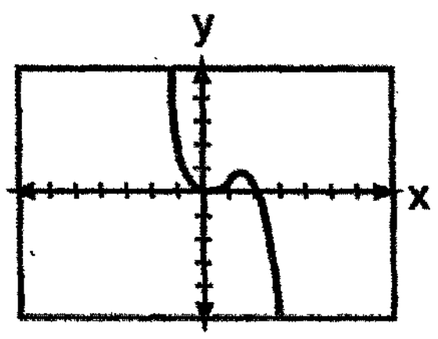
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

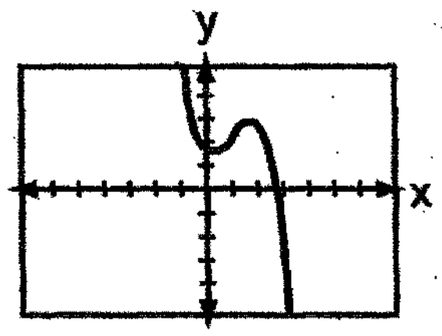
1 The accompanying graph represents the equation $y = f(x)$.



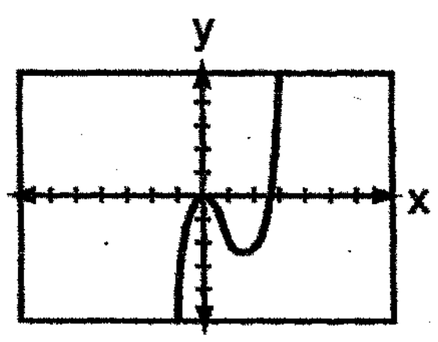
Which graph represents $g(x)$, if $g(x) = -f(x)$?



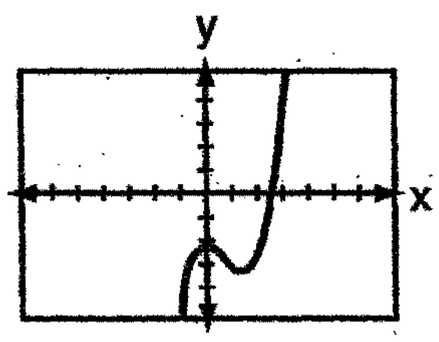
(1)



(3)



(2)



(4)

If $y = f(x)$, the equation of the reflection of f over the x-axis is $-y = f(x)$, which equals $y = -f(x)$, which equals $g(x)$.

Note, the reflection of f over the y-axis is $y = f(-x)$

2 During a single day at radio station WMZH, the probability that a particular song is played is .38. Which expression represents the probability that this song will be played on *exactly* 5 days out of 7 days?

(1) ${}_7C_5(.38)^2(.62)^5$

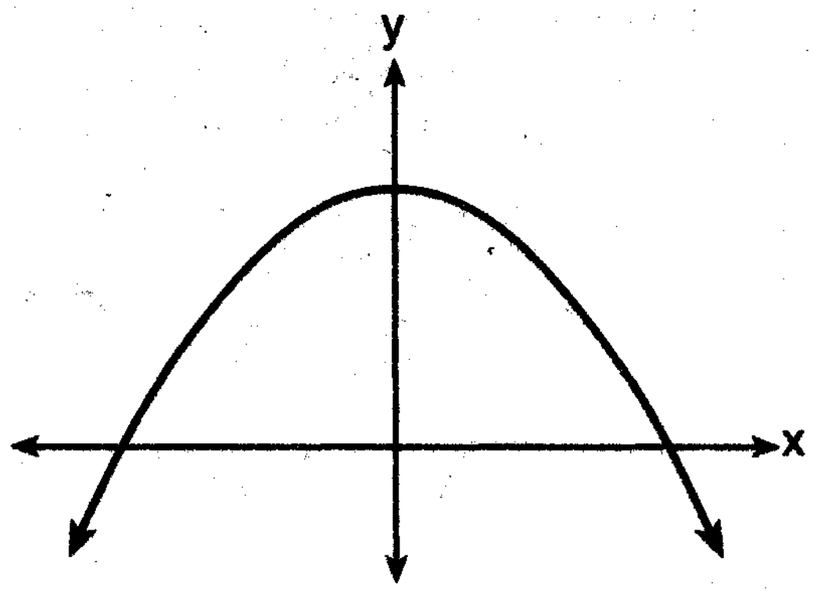
(2) ${}_7C_5(.38)^5(.62)^2$

(3) ${}_7P_5(.38)^5(.62)^2$

(4) ${}_5C_2(.38)^5(.62)^2$

Use this space for computations.

3 Which equation is best represented by the accompanying graph?



(1) $y = 6^x$
(2) $y = 6x^2$
(3) $y = 6x + 1$
(4) $y = -x^2 + 1$

the only quadratic with $a < 0$.

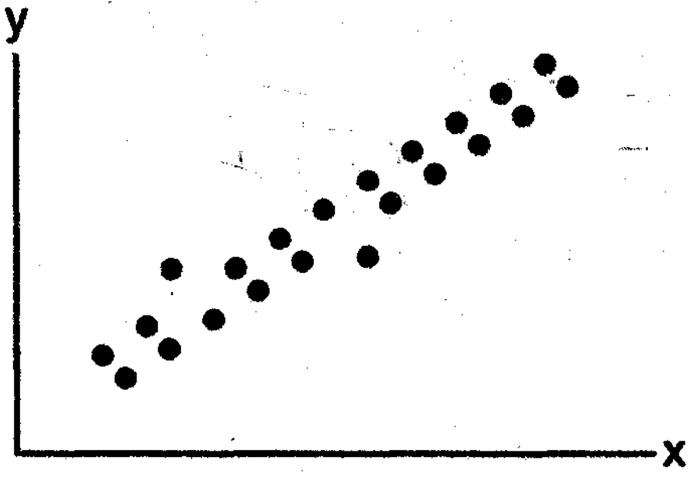
4 Jack is planting a triangular rose garden. The lengths of two sides of the plot are 8 feet and 12 feet, and the angle between them is 87°. Which expression could be used to find the area of this garden?

(1) $8 \cdot 12 \cdot \sin 87°$
(2) $8 \cdot 12 \cdot \cos 87°$
(3) $\frac{1}{2} \cdot 8 \cdot 12 \cdot \cos 87°$
(4) $\frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 87°$

$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \cdot 8 \cdot 12 \sin 87$

5 What could be the approximate value of the correlation coefficient for the accompanying scatter plot?



(1) –0.85
(2) –0.16
(3) 0.21
(4) 0.90

negative slopes

0.21 - weak correlation

Use this space for computations.

6 What is one solution of the accompanying system of equations?

$$y = -x^2 + 5$$
$$y = -0.5x^2 + 3$$

(1) (3,5)
(2) (0,5)
(3) (−2,1)
(4) (0,3)

Use substitution

$-x^2+5 = -0.5x^2+3$

$-.5x^2+5 = 3$

$\frac{-.5x^2}{-.5} = \frac{-2}{-.5}$

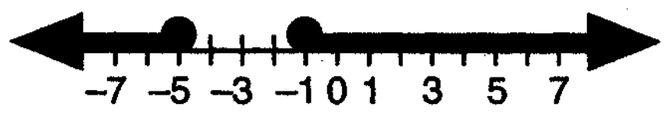
$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$y = -(-2)^2 + 5$

$y = 1$ (−2,1)

7 Which inequality is represented by the accompanying graph?



(1) $|x + 2| > 5$
(2) $|x + 3| \geq 2$
(3) $|x - 1| \leq 5$
(4) $|x - 5| \geq 2$

$|x+3| \geq 2$

$x+3 \geq 2$ or $x+3 \leq -2$

$x \geq -1$ or $x \leq -5$

8 The volume of a soap bubble is represented by the equation $V = 0.094\sqrt{A^3}$, where A represents the surface area of the bubble. Which expression is also equivalent to V?

(1) $0.094A^{\frac{3}{2}}$
(2) $0.094A^{\frac{2}{3}}$
(3) $0.094A^6$
(4) $(0.094A^3)^{\frac{1}{2}}$

$V = 0.094\sqrt{A^3}$

$V = 0.094A^{3/2}$

9 The fraction $\frac{3}{\sqrt{6}-1}$ is equivalent to

(1) $3\sqrt{6}+3$
(2) $3\sqrt{6}-3$
(3) $\frac{3\sqrt{6}+3}{5}$
(4) $\frac{3\sqrt{6}-3}{5}$

$\frac{3}{(\sqrt{6}-1)} \cdot \frac{(\sqrt{6}+1)}{(\sqrt{6}+1)} = \frac{3\sqrt{6}+3}{6-1} = \frac{3\sqrt{6}+3}{5}$

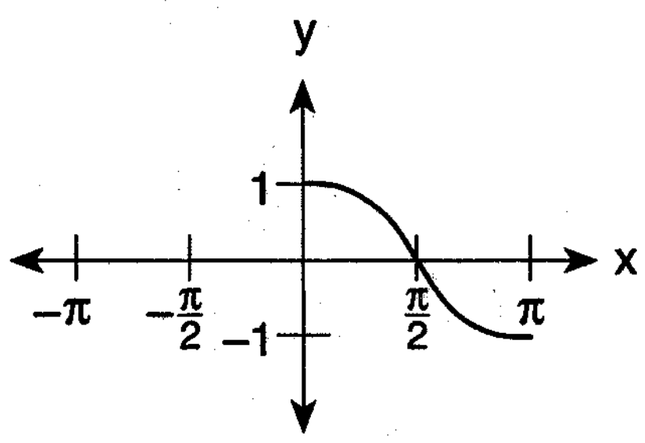
Use this space for computations.

10 A function, f, is defined by the set {(2,3), (4,7), (–1,5)}. If f is reflected in the line $y = x$, which point will be in the reflection?

(1) (5,–1)
(2) (–5,1)
(3) (1,–5)
(4) (–1,5)

The reflection in the line $y=x$ of f: {(3,2), (7,4), (5,–1)}

11 Which equation is represented by the accompanying graph?



The amplitude is 1.
The period is 2π.

(1) $y = \cos x$
(2) $y = \cos \frac{1}{2}x$
(3) $y = \cos 2x$
(4) $y = \frac{1}{2}\cos x$

12 Which expression is in simplest form?

(1) $\frac{x}{x^2} = \frac{1}{x}$
(2) $\frac{9}{x^2+9}$
(3) $\frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2} = x-2$
(4) $\frac{x^2-6x+9}{x^2-x-6}$ $\frac{(x-3)(x-3)}{(x-3)(x+2)} = \frac{x-3}{x+2}$

13 The expression $\frac{\frac{3}{x} - \frac{x}{1}}{\frac{3}{x} - 1}$ is equivalent to

(1) $\frac{1}{3}$
(2) $-\frac{1}{3}$
(3) 3
(4) –3

$\frac{\frac{3-x}{3x}}{\frac{3-x}{x}} = \frac{-1(x-3)}{3x} \cdot \frac{x}{3-x} = -\frac{1}{3}$

$1 + 2^{1/2} + 3^{1/3}$

Use this space for computations.

14 The expression $1 + \sqrt{2} + \sqrt[3]{3}$ is equivalent to

(1) $\sum_{n=1}^{3} \sqrt{n}$

(2) $\sum_{n=0}^{3} n^n$

(3) $\sum_{n=1}^{3} n^{-n}$

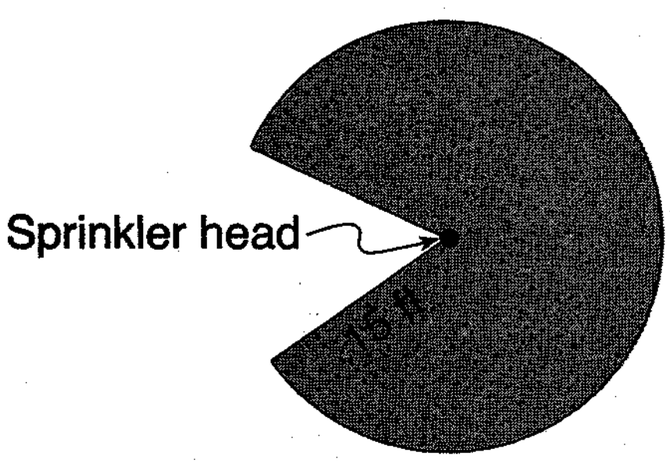
(4) $\sum_{n=1}^{3} n^{\frac{1}{n}}$

15 Which set of ordered pairs does *not* represent a function?

(1) {(3,–2), (–2,3), (4,–1), (–1,4)}
(2) {(3,–2), (3,–4), (4,–1), (4,–3)}
(3) {(3,–2), (4,–3), (5,–4), (6,–5)}
(4) {(3,–2), (5,–2), (4,–2), (–1,–2)}

The points (3,-2) & (3,-4) Fail the vertical line test.

16 Cerise waters her lawn with a sprinkler that sprays water in a circular pattern at a distance of 15 feet from the sprinkler. The sprinkler head rotates through an angle of 300°, as shown by the shaded area in the accompanying diagram.



$A = \pi r^2$

$= 15^2 \pi$

$= 225\pi \cdot \frac{300}{360}$

$= 589$

What is the area of the lawn, to the *nearest square foot*, that receives water from this sprinkler?

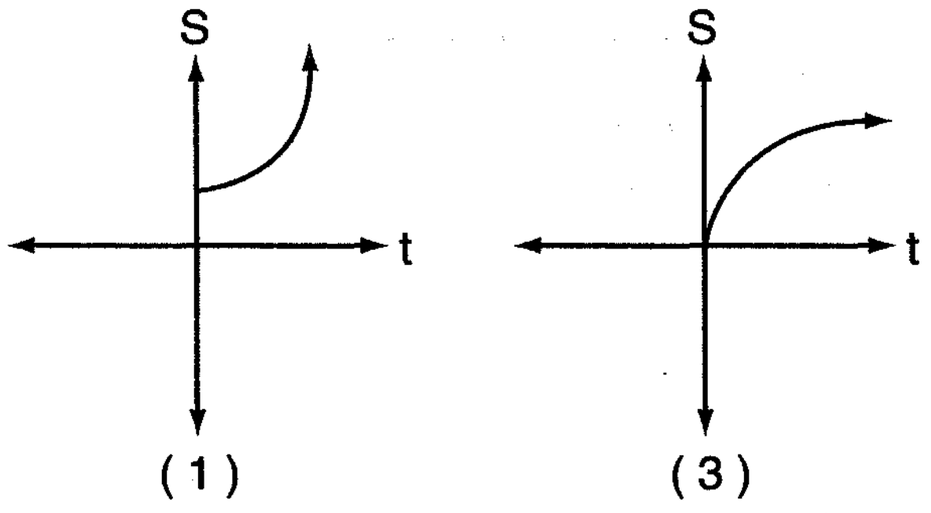
(1) 79
(2) 94
(3) 589
(4) 707

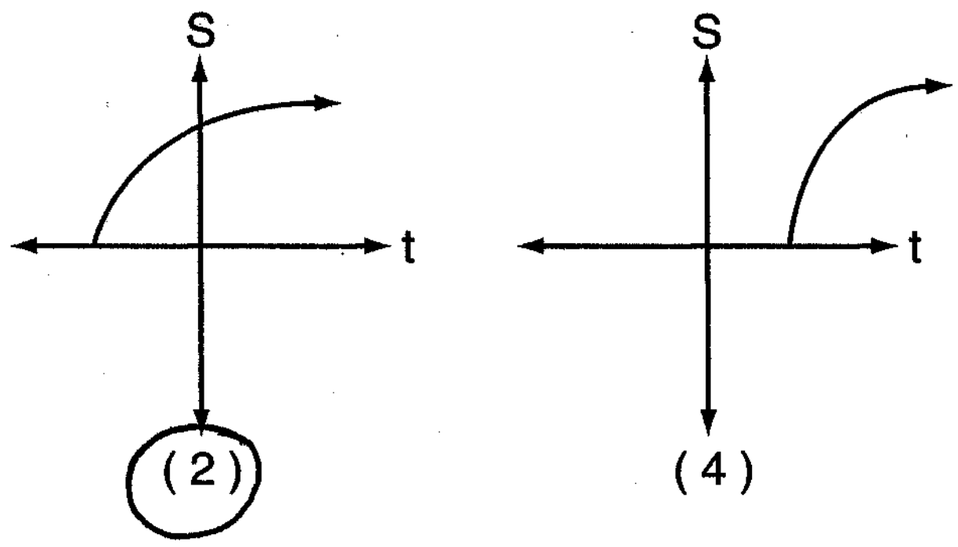
17 Which number is the discriminant of a quadratic equation whose roots are real, unequal, and irrational?

(1) 0 equal
(2) –5 imaginary
(3) 7
(4) 4 rational

Use this space for computations.

18 The formula $S = 20\sqrt{t + 273}$ is used to determine the speed of sound, S, in meters per second, near Earth's surface, where t is the surface temperature, in degrees Celsius. Which graph best represents this function?





19 If $2 + i$ and $2 - i$ are the roots of the equation $x^2 - 4x + c = 0$, what is the value of c?

(1) -5
(2) 5
(3) -4
(4) 4

$x = 2 + i \quad x = 2 - i$

$x - 2 - i = 0 \quad x - 2 + i = 0$

$(x - 2 - i)(x - 2 + i) = 0$

$x^2 - 2x + xi - 2x + 4 = 2i$

$-ix + 2i - i^2 = 0$

20 The expression $\sin A + \dfrac{\cos^2 A}{\sin A}$ is equivalent to

(1) 1
(2) $\sin A$
(3) $\sec A$
(4) $\csc A$

$x^2 - 4x + 4 - i^2 = 0$

$x^2 - 4x + 4 - (-1) = 0$

$x^2 - 4x + 5 = 0$

$\sin A + \dfrac{\cos^2 A}{\sin A}$ common denominator is $\sin A$

$\dfrac{\sin^2 A}{\sin A} + \dfrac{\cos^2 A}{\sin A} = \dfrac{\sin^2 A + \cos^2 A}{\sin A} = \dfrac{1}{\sin A} = \csc A$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 A population of wolves in a county is represented by the equation $P(t) = 80(0.98)^t$, where t is the number of years since 1998. Predict the number of wolves in the population in the year 2008.

2008 - 1998 = 10

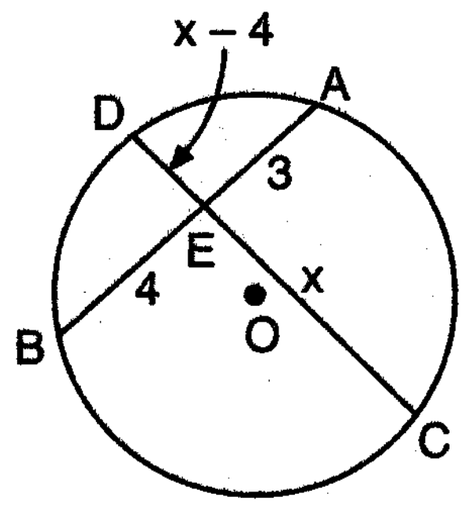
$P(10) = 80(0.98)^{10} = \boxed{65}$

22 The accompanying table shows the enrollment of a preschool from 1980 through 2000. Write a linear regression equation to model the data in the table.

Year (x)	Enrollment (y)
1980	14
1985	20
1990	22
1995	28
2000	37

$\boxed{Y = 1.08x - 2125}$

23 In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x - 4$, what is the value of x?



$(\overline{AE})(\overline{BE}) = (\overline{CE})(\overline{DE})$

$3 \cdot 4 = x(x-4)$

$12 = x^2 - 4x$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\boxed{x = 6}$ or ~~$x = -2$~~ substituting -2 results in negative length

24 Denise is designing a storage box in the shape of a cube. Each side of the box has a length of 10 inches. She needs more room and decides to construct a larger box in the shape of a cube with a volume of 2,000 cubic inches. By how many inches, to the *nearest tenth*, should she *increase* the length of each side of the original box?

$(x+10)^3 = 2000$

take cubic root of each side

$x + 10 = \sqrt[3]{2000}$

$x + 10 = 12.6$

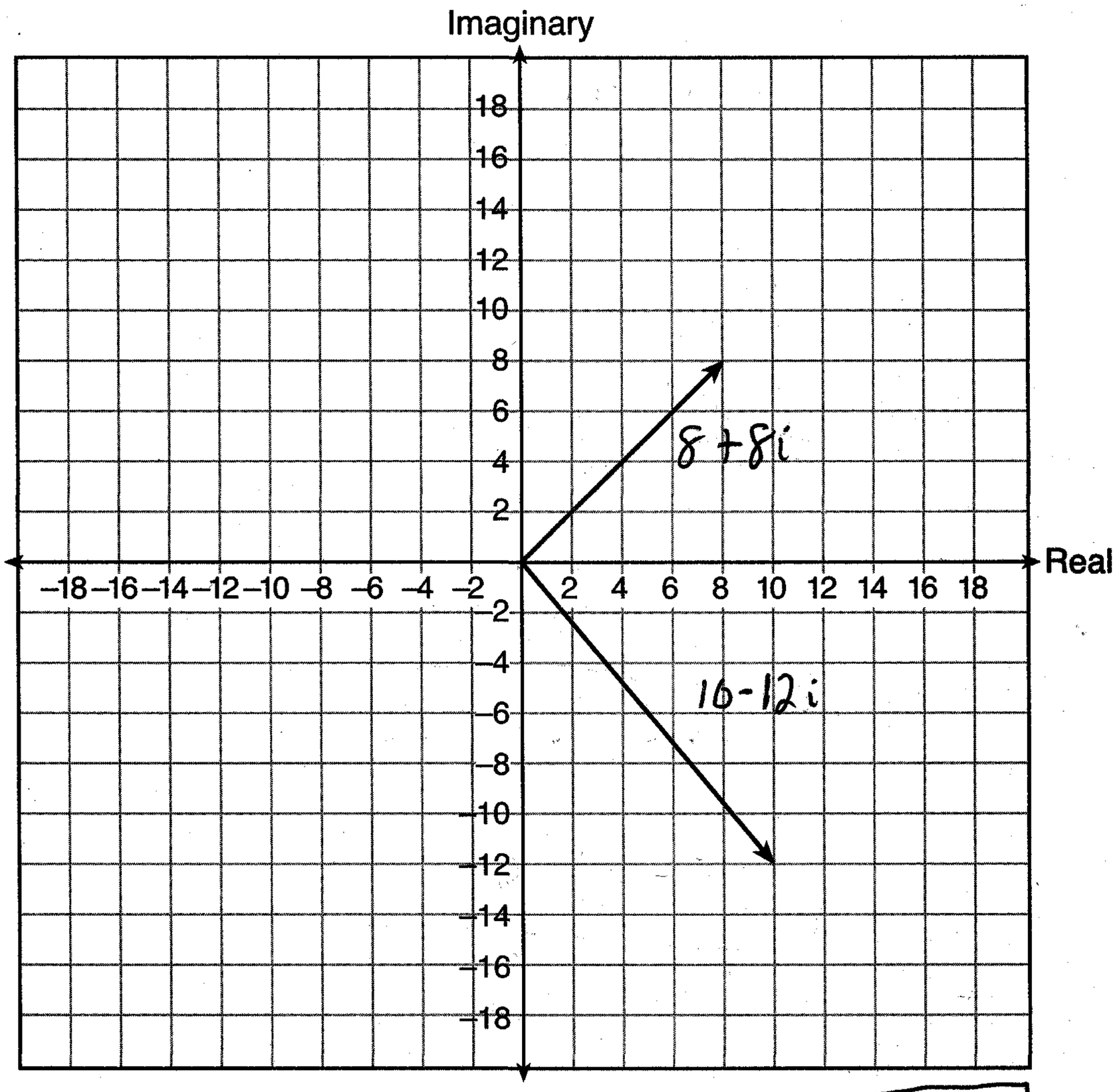
$\boxed{x = 2.6}$

25 If $f(x) = \log_2 x$ and $g(x) = 2x^2 + 14$, determine the value of $(f \circ g)(5)$.

$g(5) = 2(5)^2 + 14 = 64$

$F(64) = \log_2 64 = \boxed{6}$

26 On a stamp honoring the German mathematician Carl Gauss, several complex numbers appear. The accompanying graph shows two of these numbers. Express the sum of these numbers in $a + bi$ form.



$(8+8i) + (10-12i) = \boxed{18-4i}$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 If $f(x) = \frac{3x^2 - 27}{18x + 30}$ and $g(x) = \frac{x^2 - 7x + 12}{3x^2 - 7x - 20}$, find $f(x) \div g(x)$ for all values of x for which the expression is defined and express your answer in simplest form.

$f(x) \div g(x)$

$\frac{3x^2-27}{18x+30} \div \frac{x^2-7x+12}{3x^2-7x-20}$

$\frac{3x^2-27}{18x+30} \times \frac{3x^2-7x-20}{x^2-7x+12}$

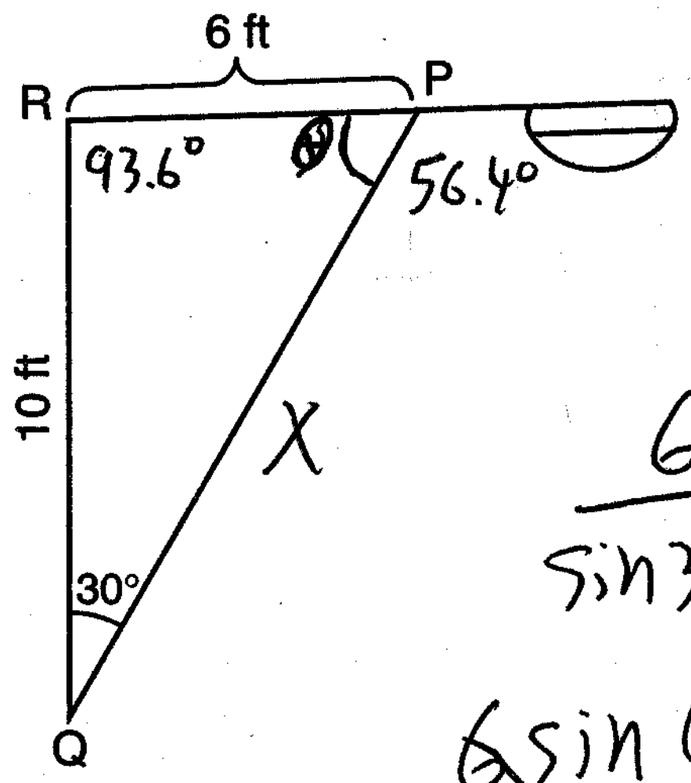
$\frac{3(x^2-9)}{6(3x+5)} \times \frac{(3x+5)(x-4)}{(x-3)(x-4)}$

$\frac{(x+3)(x-3)}{2} \times \frac{1}{x-3}$

$\boxed{\frac{x+3}{2}}$

28 In the accompanying diagram of a streetlight, the light is attached to a pole at R and supported by a brace, $\overline{PQ}$, $RQ = 10$ feet, $RP = 6$ feet, $\angle PRQ$ is an obtuse angle, and $m\angle PQR = 30$.

Find the length of the brace, $\overline{PQ}$, to the *nearest foot*.



Law of Sines

$$\frac{6}{\sin 30} = \frac{10}{\sin\theta}$$

$$\frac{6\sin\theta}{6} = \frac{10\sin 30}{6}$$

$$\sin\theta = \frac{5}{6}$$

$$\theta = 56.4°$$

$m\angle PRQ = 180 - 56.4 - 30 = 93.6°$

Law of Sines

$$\frac{x}{\sin 93.6} = \frac{6}{\sin 30}$$

$$\boxed{x = 12}$$

29 Conant High School has 17 students on its championship bowling team. Each student bowled one game. The scores are listed in the accompanying table.

Score (x_i)	Frequency (f_i)
140	4
145	3
150	2
160	3
170	2
180	2
194	1

Find, to the *nearest tenth*, the population standard deviation of these scores. 16.2

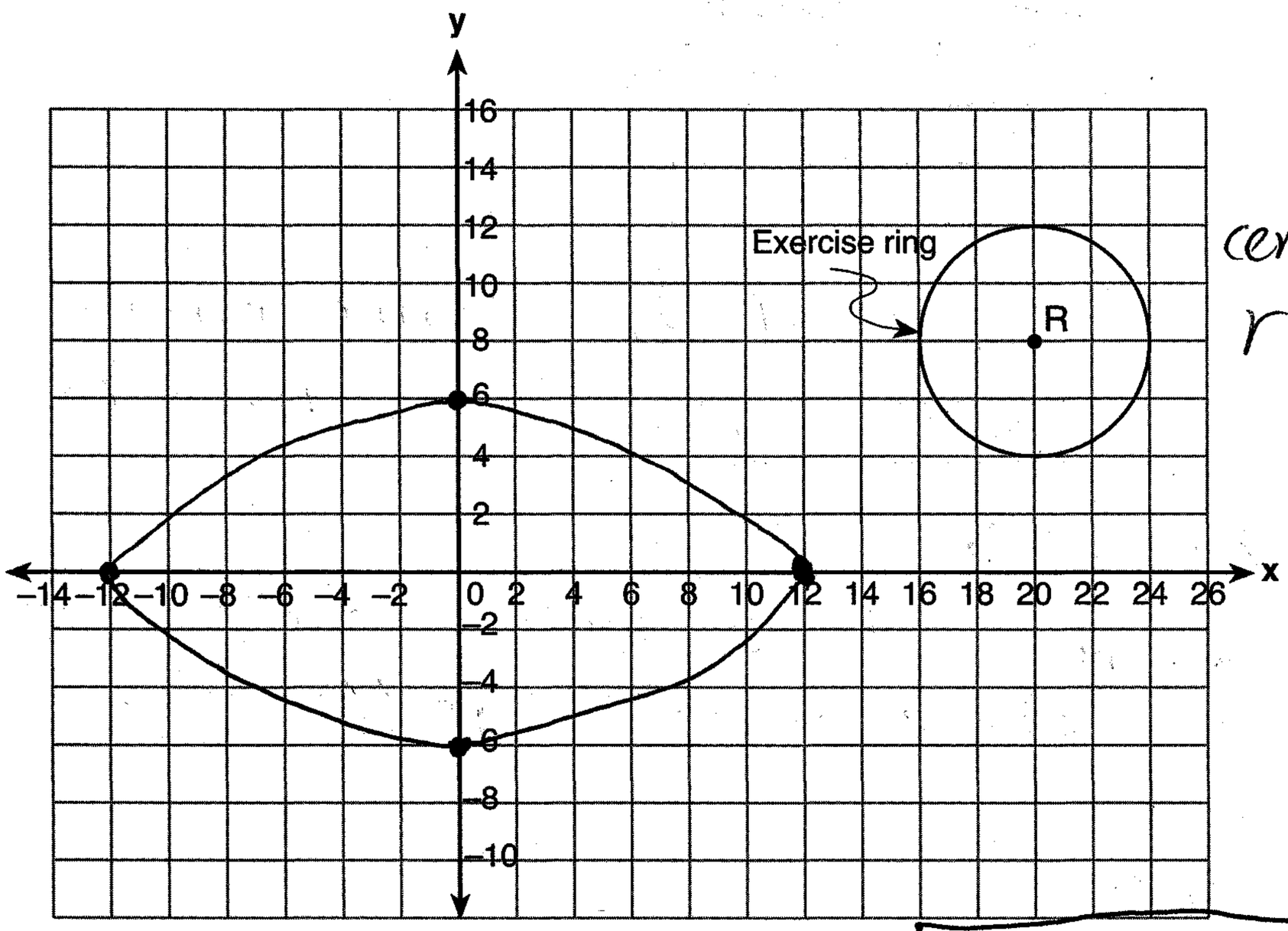
How many of the scores fall within one standard deviation of the mean?

$\bar{x} = 157$

The relevant range is 140.8 - 173.2

Ten scores fall within this range.

30 A landscape architect is working on the plans for a new horse farm. He is laying out the exercise ring and racetrack on the accompanying graph. The location of the circular exercise ring, with point R as its center, has already been plotted.



center (20, 8)
r = 4

Write an equation that represents the outside edge of the exercise ring. $(x-20)^2+(y-8)^2=16$

The equation of the outside edge of the racetrack is $\frac{x^2}{144}+\frac{y^2}{36}=1$.

Sketch the outside edge of the racetrack on the graph.

The semi major axis is 12

The semi minor axis is 6

31 The average annual snowfall in a certain region is modeled by the function $S(t) = 20 + 10\cos\left(\frac{\pi}{5}t\right)$, where S represents the annual snowfall, in inches, and t represents the number of years since 1970.

What is the minimum annual snowfall, in inches, for this region?

In which years between 1970 and 2000 did the minimum amount of snow fall? [The use of the grid on the next page is optional.]

The y intercept of the cosine function is (0,1). When a = 10, the amplitude is 10 & the y intercept is (0,10).

The 20 represents a positive vertical shift of 20, & the y-intercept is (0, 30).

The minimum is 10

$$10 = 20 + 10\cos\left(\frac{\pi}{5}t\right)$$
$$-10 = 10\cos\left(\frac{\pi}{5}t\right)$$
$$-1 = \cos\left(\frac{\pi}{5}t\right)$$
$$\frac{\pi}{5}t = \cos^{-1}-1$$

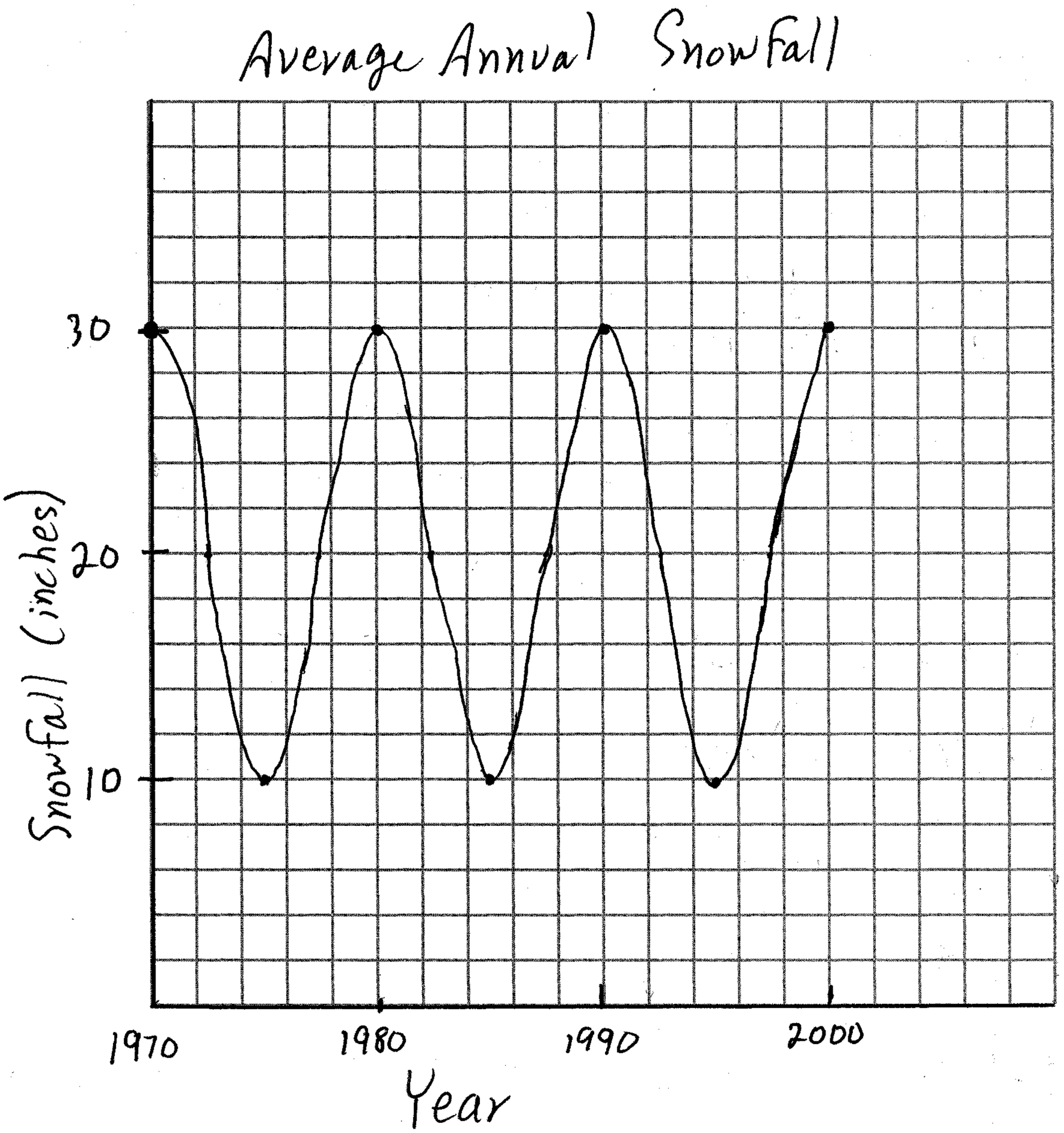
The domain is $0 < t < 30$

$\frac{\pi}{5}t = \pi$ $\quad$ $\frac{\pi}{5}t = 3\pi$ $\quad$ $\frac{\pi}{5}t = 5\pi$

$\frac{t}{5} = \frac{\pi}{\pi}$ $\quad$ $t = 15$ $\quad$ $t = 25$

$t = 5$ (1975) $\quad$ (1985) $\quad$ (1995)

Question 31 continued



32 The path of a rocket fired during a fireworks display is given by the equation $s(t) = 64t - 16t^2$, where t is the time, in seconds, and s is the height, in feet.

What is the maximum height, in feet, the rocket will reach?

In how many seconds will the rocket hit the ground? [The use of the grid on the next page is optional.]

Axis of symmetry

$x = \frac{-b}{2a} = \frac{-64}{2(-16)} = 2$ seconds

$x = 64(2) - 16(4) =$ 64 feet

$64t - 16t^2 = 0$

$16t(4 - t) = 0$

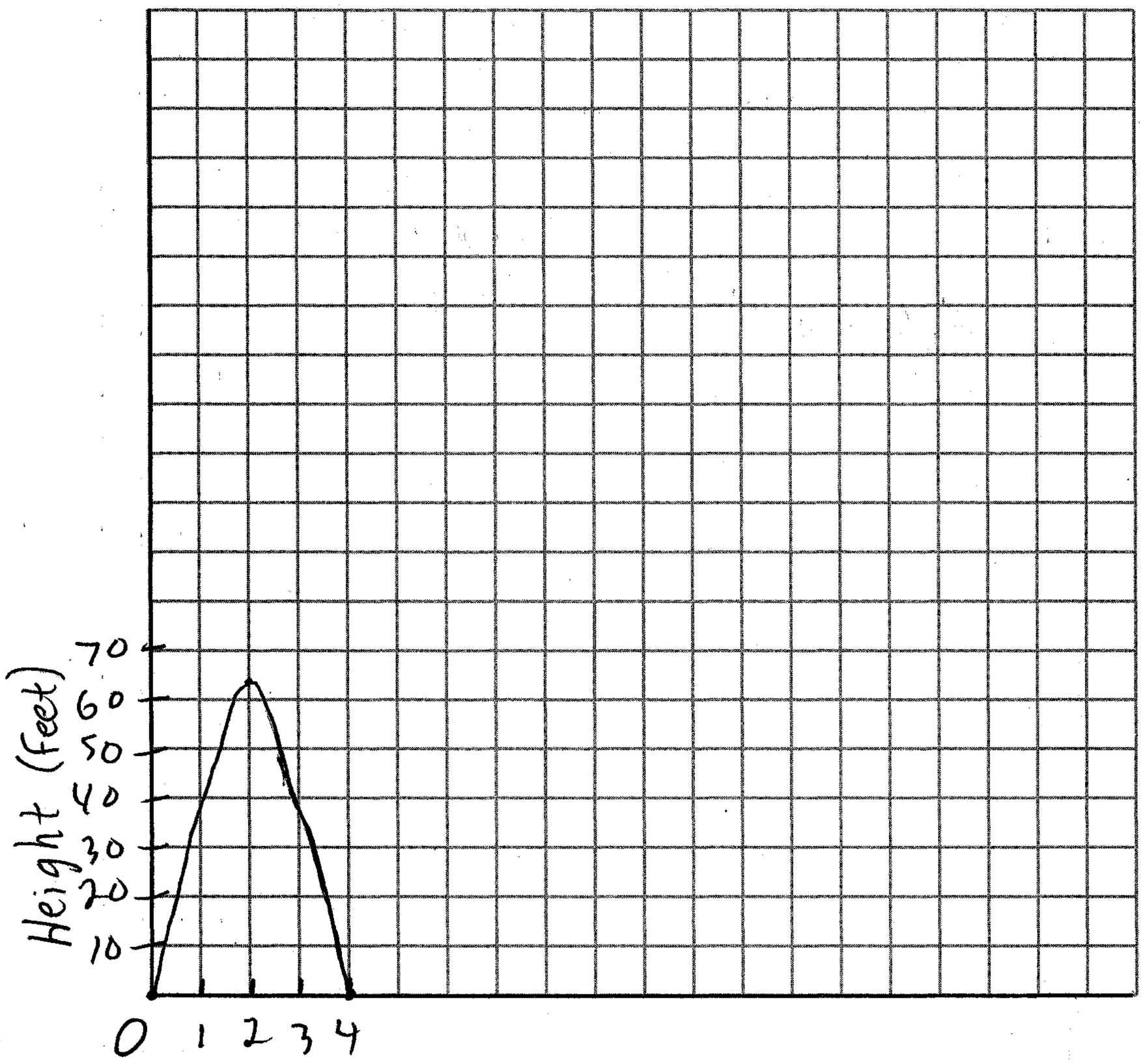
$\frac{16t}{16} = \frac{0}{16}$ $\quad 4 - t = 0$

$t = 0$ $\quad t = 4$

The rocket is on the ground at launch.

The rocket will hit the ground in 4 seconds.

Question 32 continued



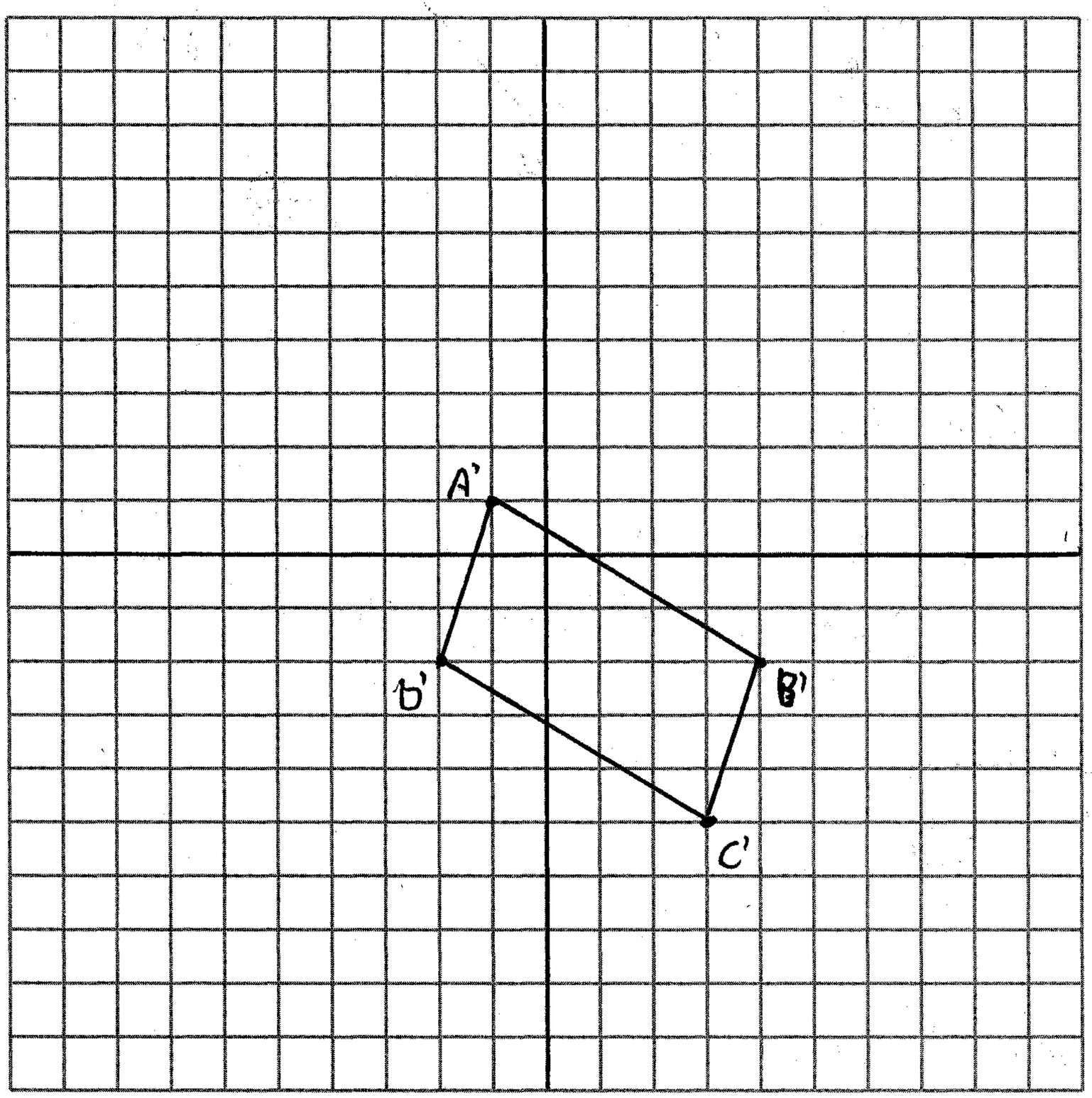
Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Given: quadrilateral $ABCD$ with vertices $A(-2,2)$, $B(8,-4)$, $C(6,-10)$, and $D(-4,-4)$. State the coordinates of $A'B'C'D'$, the image of quadrilateral $ABCD$ under a dilation of factor $\frac{1}{2}$. Prove that $A'B'C'D'$ is a parallelogram. [The use of the grid on the next page is optional.]

The coordinates of A'B'C'D' are
A'(-1, 1), B'(4, -2), C'(3, -5), D'(-2, -2)

Question 33 continued

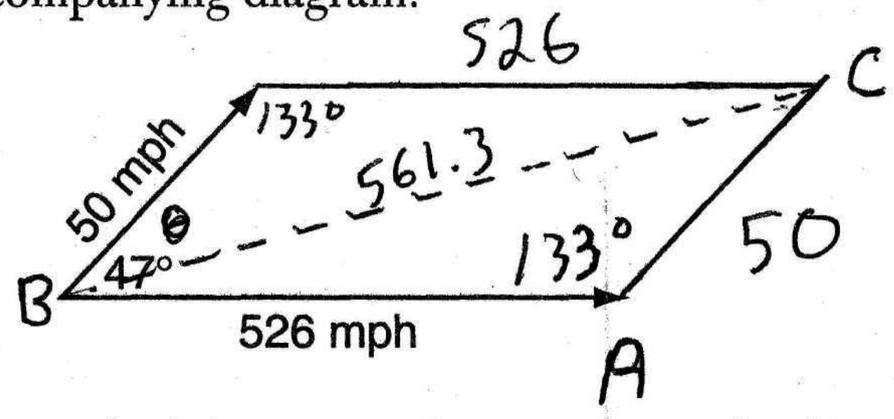


slope of $\overline{A'B'}$ is $\frac{1-(-2)}{-1-4} = \frac{-3}{5}$ slope of $\overline{B'C'}$ is $\frac{-2-(-5)}{4-3} = 3$

slope of $\overline{C'D'}$ is $\frac{-5-(-2)}{3-(-2)} = \frac{-3}{5}$ slope of $\overline{A'D'}$ is $\frac{1-(-2)}{-1-(-2)} = 3$

Opposite sides have equal slope.
Therefore opposite sides are parallel and
A'B'C'D' is a parallelogram.

34 A jet is flying at a speed of 526 miles per hour. The pilot encounters turbulence due to a 50-mile-per-hour wind blowing at an angle of 47°, as shown in the accompanying diagram.



$180° - 47° = 133°$

Find the resultant speed of the jet, to the *nearest tenth of a mile per hour*.

Use this answer to find the measure of the angle between the resultant force and the wind vector, to the *nearest tenth of a degree*.

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 50^2 + 526^2 - 2(50)(526) \cos 133°$

$a^2 = 315,049$

$a = \boxed{561.3}$

Law of Sines

$\frac{526}{\sin\theta} = \frac{561.3}{\sin 133}$

$\sin\theta = .685$

$\theta = 43.3$