The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 20, 2002 — 1:15 to 4:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 2.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

Use this space for computations.

1 What is the value of $\sum_{m=2}^{5}(m^2 - 1)$?

(1) 58 (3) 53
(2) 54 (4) 50

m	m^2-1
2	3
3	8
4	15
5	24
	50

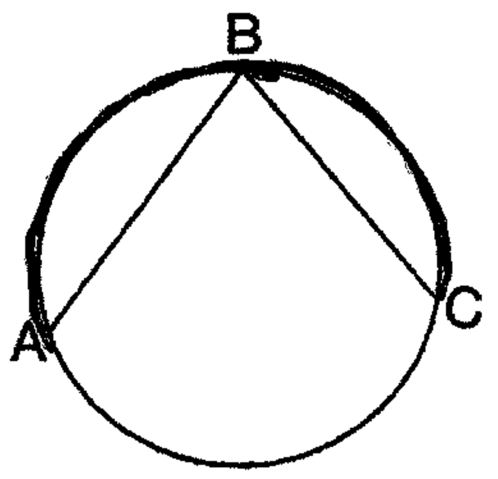
2 For all values of x for which the expression is defined, $\frac{2x + x^2}{x^2 + 5x + 6}$ is equivalent to

(1) $\frac{1}{x+3}$ (3) $\frac{1}{x+2}$

(2) $\frac{x}{x+3}$ (4) $\frac{x}{x+2}$

$= \frac{x(\cancel{2+x})}{(\cancel{x+2})(x+3)} = \frac{x}{x+3}$

3 In the accompanying diagram, the length of $\widehat{ABC}$ is $\frac{3\pi}{2}$ radians.



(Not drawn to scale)

$\frac{3\cancel{\pi}}{2} \cdot \frac{180°}{\cancel{\pi}} = 270° = \widehat{ABC}$

$90 = \widehat{AC}$

$m\angle ABC = \frac{1}{2}(90)$

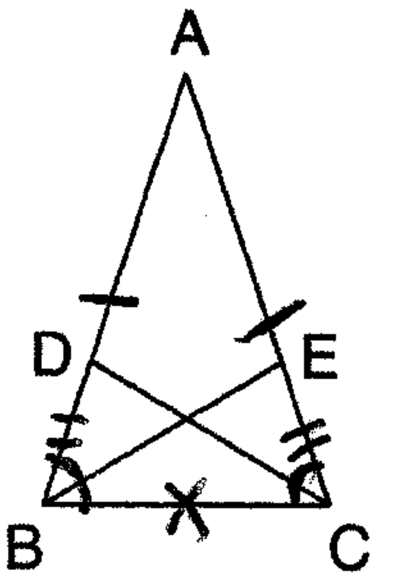
$= 45$

What is m$\angle ABC$?

(1) 36 (3) 53
(2) 45 (4) 72

Use this space for computations.

4 In the accompanying diagram of $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{BD} = \frac{1}{3}\overline{BA}$, and $\overline{CE} = \frac{1}{3}\overline{CA}$.



Triangle *EBC* can be proved congruent to triangle *DCB* by

(1) SAS ≅ SAS
(2) ASA ≅ ASA
(3) SSS ≅ SSS
(4) HL ≅ HL

5 The path of a rocket is represented by the equation $y = \sqrt{25 - x^2}$. The path of a missile designed to intersect the path of the rocket is represented by the equation $x = \frac{3}{2}\sqrt{y}$. The value of x at the point of intersection is 3. What is the corresponding value of y?

(1) −2
(2) 2
(3) −4
(4) 4

If $x = 3$

$y = \sqrt{25 - x^2}$ $\quad x = \frac{3}{2}\sqrt{y}$

$y = \sqrt{25 - 3^2}$ $\quad 3 = \frac{3}{2}\sqrt{y}$

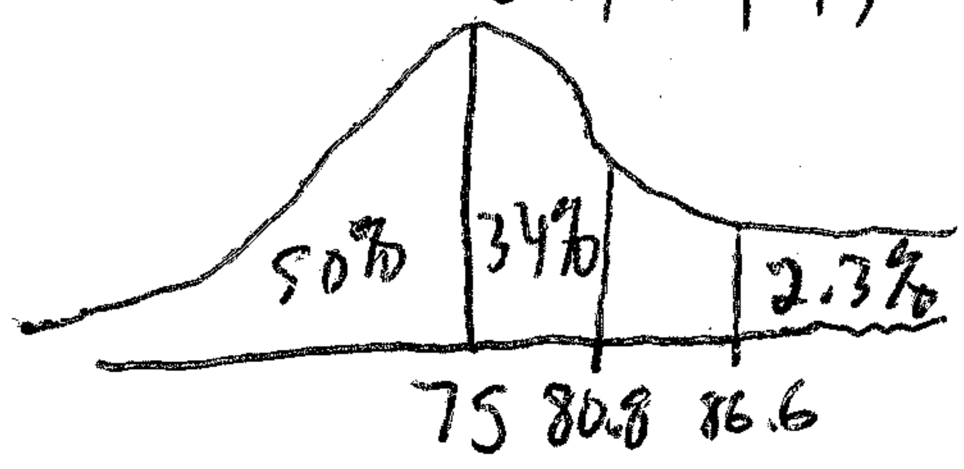
$y = 4$ $\quad 2 = \sqrt{y}$

$4 = y$

6 On a standardized test, the distribution of scores is normal, the mean of the scores is 75, and the standard deviation is 5.8. If a student scored 83, the student's score ranks

(1) below the 75th percentile
(2) between the 75th percentile and the 84th percentile
(3) between the 84th percentile and the 97th percentile
(4) above the 97th percentile

84 | 97



7 Which statement is true for all real number values of x?

(1) $|x - 1| > 0$
(2) $|x - 1| > (x - 1)$
(3) $\sqrt{x^2} = x$
(4) $\sqrt{x^2} = |x|$

(1) and (2) are false if $x > 0$
(3) is false if x is negative

Use this space for computations.

8 If x is a positive integer, $4x^{\frac{1}{2}}$ is equivalent to

(1) $\frac{2}{x}$ (3) $4\sqrt{x}$

(2) $2x$ (4) $4\frac{1}{x}$

9 What is the equation of a parabola that goes through points (0,1), (–1,6), and (2,3)?

(1) $y = x^2 + 1$ (3) $y = x^2 - 3x + 1$

(2) $y = 2x^2 + 1$ (4) $y = 2x^2 - 3x + 1$

10 If $f(x) = 2x^2 + 4$ and $g(x) = x - 3$, which number satisfies $f(x) = (f \circ g)(x)$?

(1) $\frac{3}{2}$ (3) 5

(2) $\frac{3}{4}$ (4) 4

$2x^2 \cancel{+4} = 2(x-3)^2 \cancel{+4}$

$2x^2 = 2(x^2 - 6x + 9)$

$\cancel{2x^2} = \cancel{2x^2} - 12x + 18$

$\frac{12x}{12} = \frac{18}{12} = \frac{3}{2}$

11 A linear regression equation of best fit between a student's attendance and the degree of success in school is $h = 0.5x + 68.5$. The correlation coefficient, r, for these data would be

(1) $0 < r < 1$ (3) $r = 0$

(2) $-1 < r < 0$ (4) $r = -1$

Because the slope is positive, the correlation coefficient is positive

12 What is the solution set of the equation $\frac{x}{x-4} - \frac{1}{x+3} = \frac{28}{x^2 - x - 12}$?

(1) { } (3) {–6}

(2) {4,–6} (4) {4}

$\frac{x(x+3) - (x-4)}{(x-4)(x+3)} = \frac{28}{(x-4)(x+3)}$

$x^2 + 3x - x + 4 = 28$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6$ $\boxed{x = 4}$ extraneous because of

$\frac{4}{\boxed{4-4}} - \frac{1}{4+3} = \frac{28}{\boxed{4^2-4-12}}$

13 Which equation represents a function?

(1) $4y^2 = 36 - 9x^2$ ellipse (3) $x^2 + y^2 = 4$ circle

(2) $y = x^2 - 3x - 4$ (4) $x = y^2 - 6x + 8$

Use this space for computations.

14 What is the solution set of the equation $x = 2\sqrt{2x-3}$?

(1) { } (3) {6}

(2) {2} (4) {2,6}

$x = 2\sqrt{2x-3}$

$x^2 = 4(2x-3)$

$x^2 = 8x-12$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

6 2

15 What is the sum of $\sqrt{-2}$ and $\sqrt{-18}$?

(1) $5i\sqrt{2}$ (3) $2i\sqrt{5}$

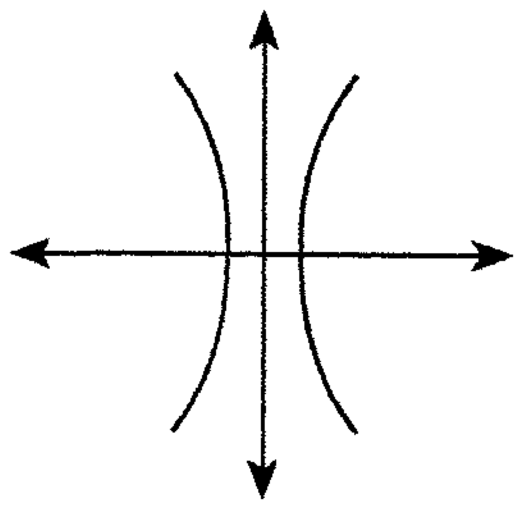
(2) $4i\sqrt{2}$ (4) $6i$

$\sqrt{2}\,i + \sqrt{18}\,i$

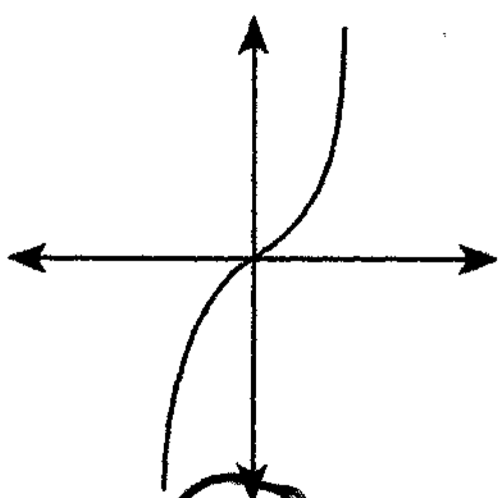
$\sqrt{2}\,i + \sqrt{9}\sqrt{2}\,i$

$\sqrt{2}\,i + 3\sqrt{2}\,i$

16 Which diagram represents a one-to-one function?

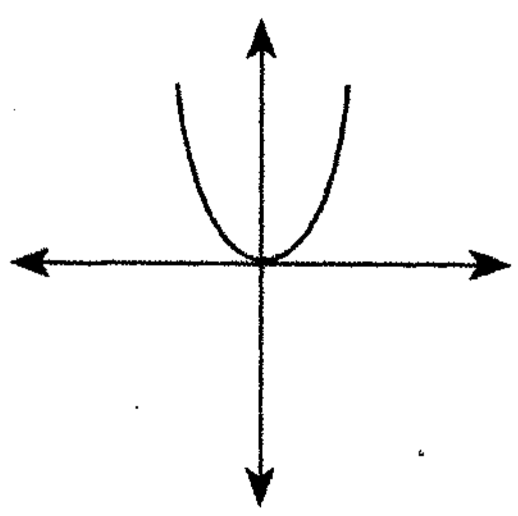


(1)

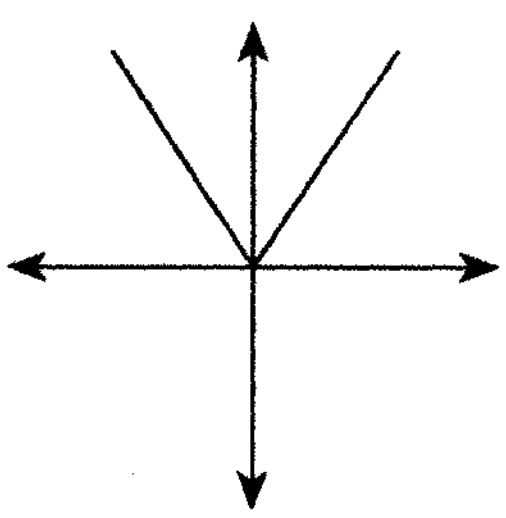


(3)

passes the horizontal line test.



(2)



(4)

17 Point P' is the image of point $P(-3,4)$ after a translation defined by $T_{(7,-1)}$. Which other transformation on P would also produce P'?

Use this space for computations.

(1) $r_{y=-x}$
(2) $r_{y\text{-axis}}$
(3) $R_{90°}$
(4) $R_{-90°}$

After the translation, $P'(4,3)$

$R_{-90°}$ is represented as $(y,-x)$

18 Which transformation does *not* preserve orientation?

(1) translation
(2) dilation
(3) reflection in the y-axis
(4) rotation

19 The roots of the equation $2x^2 - x = 4$ are

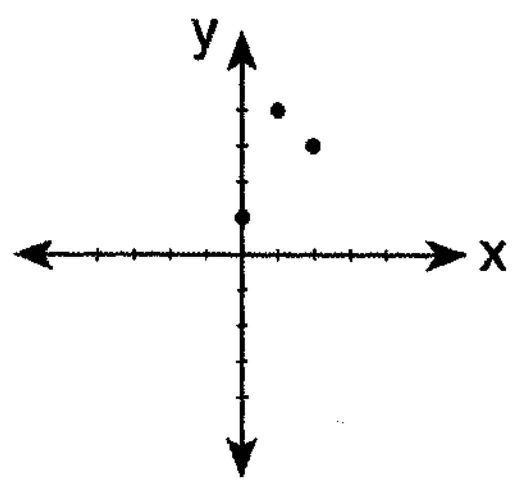
(1) real and irrational
(2) real, rational, and equal
(3) real, rational, and unequal
(4) imaginary

$2x^2 - x - 4 = 0$

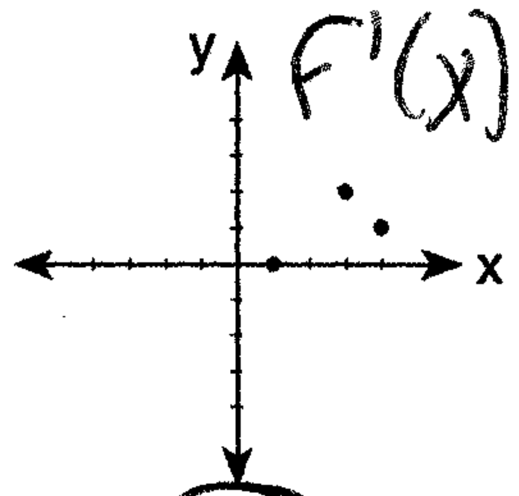
$b^2 - 4ac$
$(-1)^2 - 4(2)(-4)$
$1 + 32$
33

20 Which graph represents the inverse of $f(x) = \{(0,1),(1,4),(2,3)\}$?

$f^{-1}(x) = \{(1,0),(4,1),(3,2)\}$



(1)



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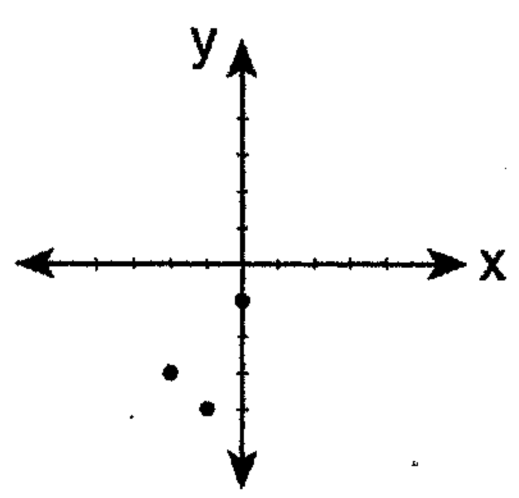

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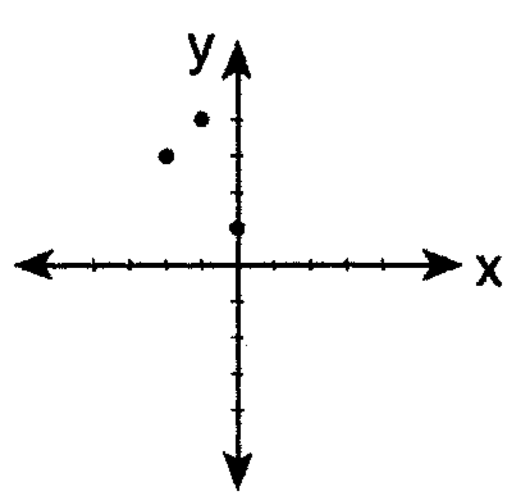


(4)

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 On a nationwide examination, the Adams School had a mean score of 875 and a standard deviation of 12. The Boswell School had a mean score of 855 and a standard deviation of 20. In which school was there greater consistency in the scores? Explain how you arrived at your answer.

The was greater consistency in the scores at Adams since their scores had a lower standard deviation.

22 Is $\frac{1}{2}$ sin $2x$ the same expression as sin x? Justify your answer.

No. If $x = 90°$:

$\frac{1}{2} \sin 2(90) = 0$

$\sin 90 = 1$

23 After studying a couple's family history, a doctor determines that the probability of any child born to this couple having a gene for disease X is 1 out of 4. If the couple has three children, what is the probability that *exactly* two of the children have the gene for disease X?

$n=3$

$r=2$

$p=\frac{1}{4}$

$q=\frac{3}{4}$

$_nC_r\, p^r\, q^{n-r}$

$_3C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1$

$3\left(\frac{1}{16}\right)\left(\frac{3}{4}\right)=\frac{9}{64}$

24 Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$, where:

G = final number of bacteria
A = initial number of bacteria
t = time (in hours)

In approximately how many hours will 4 bacteria first increase to 2,500 bacteria? Round your answer to the *nearest hour*.

$G = A(2.7)^{0.584t}$

$\frac{2500}{4} = \frac{\cancel{4}(2.7)^{0.584t}}{\cancel{4}}$

$\log 625 = \log 2.7^{0.584t}$

$\frac{\log 625}{0.584 \log 2.7} = \frac{\cancel{0.584}t \cdot \cancel{\log 2.7}}{\cancel{0.584 \log 2.7}}$

$11.09 \approx t$

The bacteria do not increase to 2,500 until after the 11th hour. The answer is 12 hours.

25 The equation $W = 120I - 12I^2$ represents the power (W), in watts, of a 120-volt circuit having a resistance of 12 ohms when a current (I) is flowing through the circuit. What is the maximum power, in watts, that can be delivered in this circuit?

To find the maximum, find the axis of symmetry of $W = -12I^2 + 120I$

$$I = x = \frac{-B}{2A} = \frac{-120}{2(-12)} = 5$$

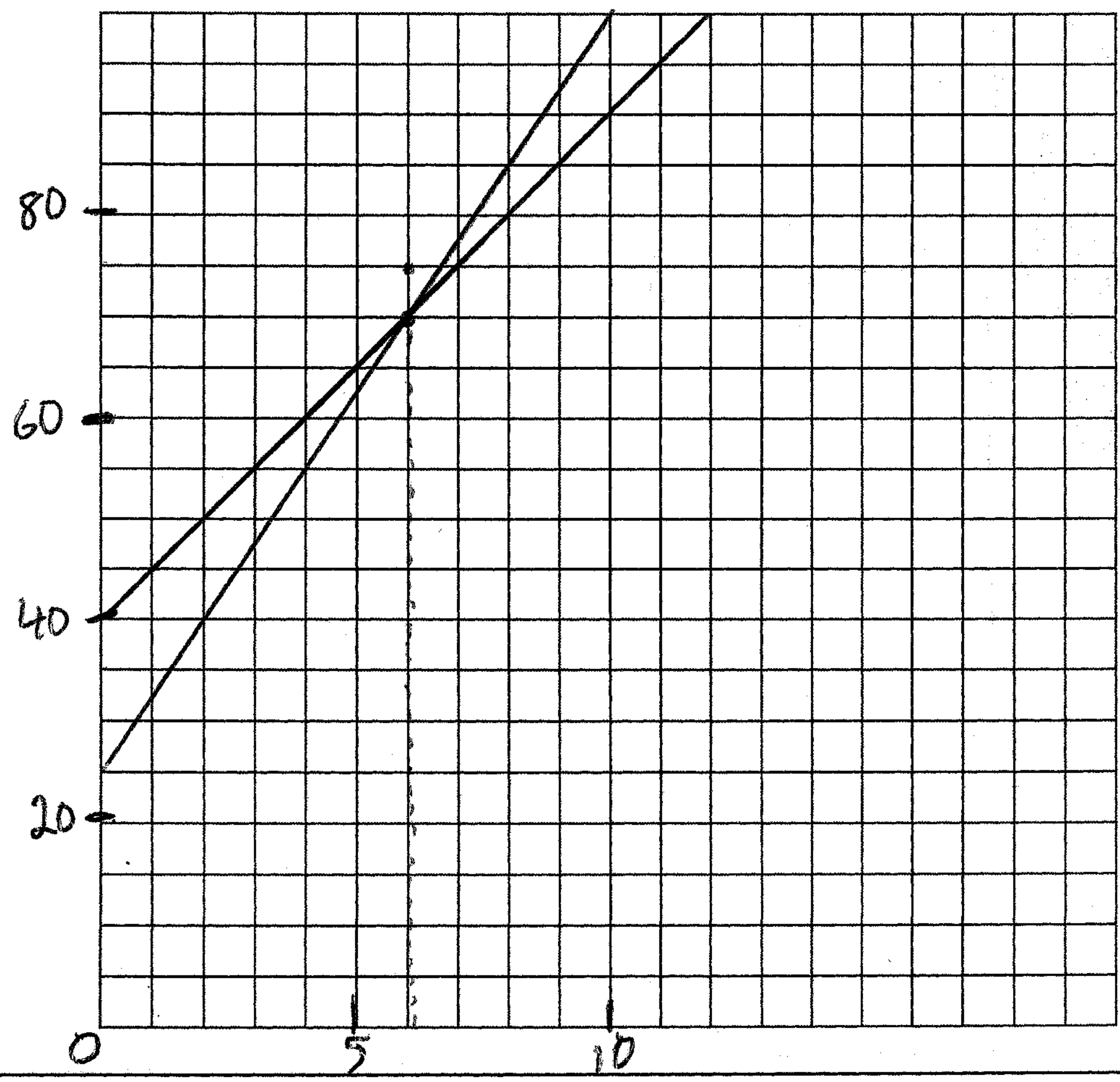
$$W = 120I - 12I^2$$
$$= 120(5) - 12(5)^2$$
$$= 300$$

26 Island Rent-a-Car charges a car rental fee of \$40 plus \$5 per hour or fraction of an hour. Wayne's Wheels charges a car rental fee of \$25 plus \$7.50 per hour or fraction of an hour. Under what conditions does it cost *less* to rent from Island Rent-a-Car? [The use of the accompanying grid is optional.]

$25 + 7.5h > 40 + 5h$

$2.5h > 15$

$h > 6$



Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 An electronics company produces a headphone set that can be adjusted to accommodate different-sized heads. Research into the distance between the top of people's heads and the top of their ears produced the following data, in inches:

4.5, 4.8, 6.2, 5.5, 5.6, 5.4, 5.8, 6.0, 5.8, 6.2, 4.6, 5.0, 5.4, 5.8

The company decides to design their headphones to accommodate three standard deviations from the mean. Find, to the *nearest tenth,* the mean, the standard deviation, and the range of distances that must be accommodated.

$\bar{X} \approx 5.5$

$\sigma_X \approx .5$

$3\sigma_X = 1.5$

range is 4-7 inches

28 A pelican flying in the air over water drops a crab from a height of 30 feet. The distance the crab is from the water as it falls can be represented by the function $h(t) = -16t^2 + 30$, where t is time, in seconds. To catch the crab as it falls, a gull flies along a path represented by the function $g(t) = -8t + 15$. Can the gull catch the crab before the crab hits the water? Justify your answer. [The use of the accompanying grid is optional.]

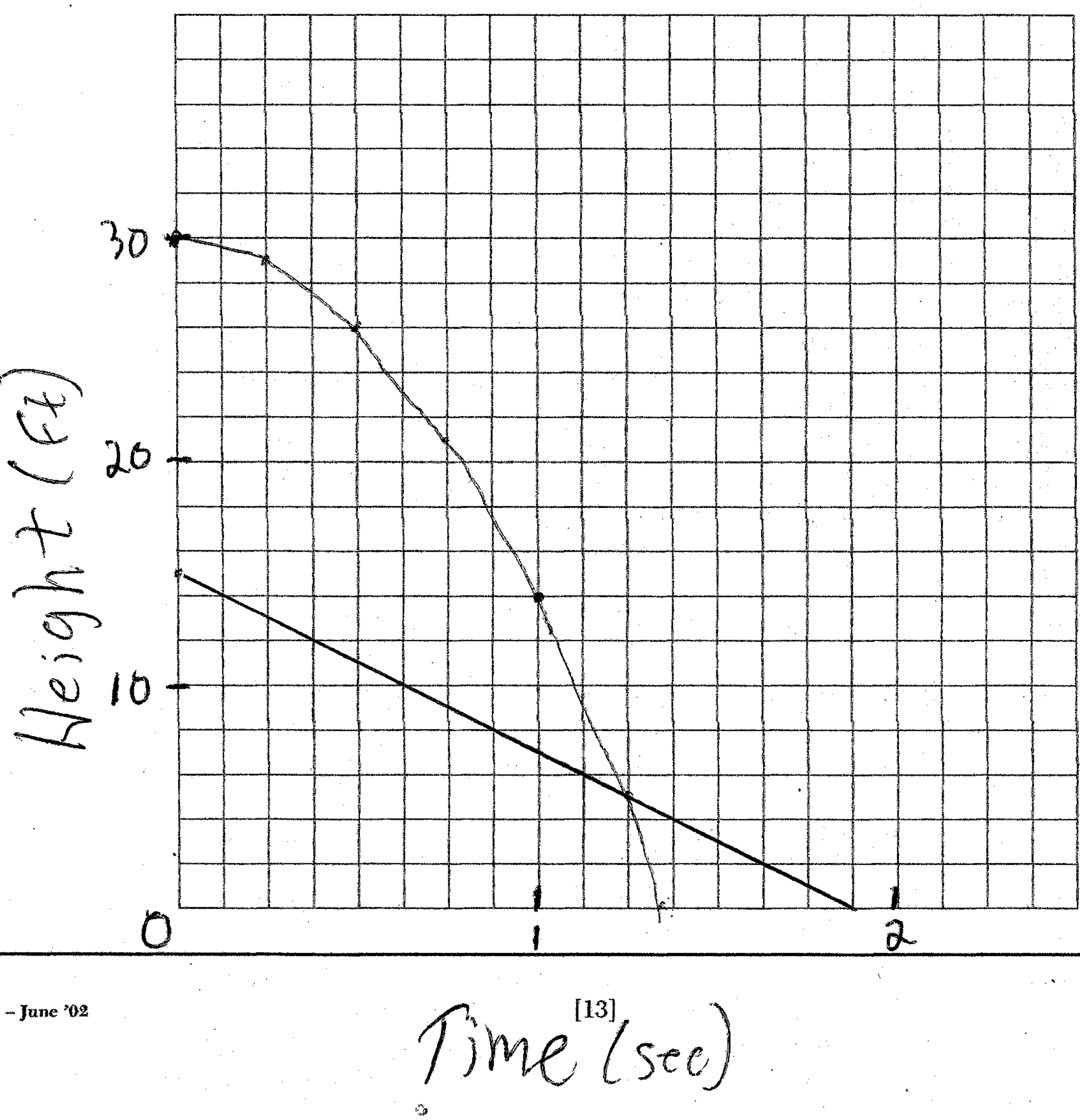
$-16t^2+30=-8t+15$

$-16t^2+8t+15=0$

$16t^2-8t-15=0$

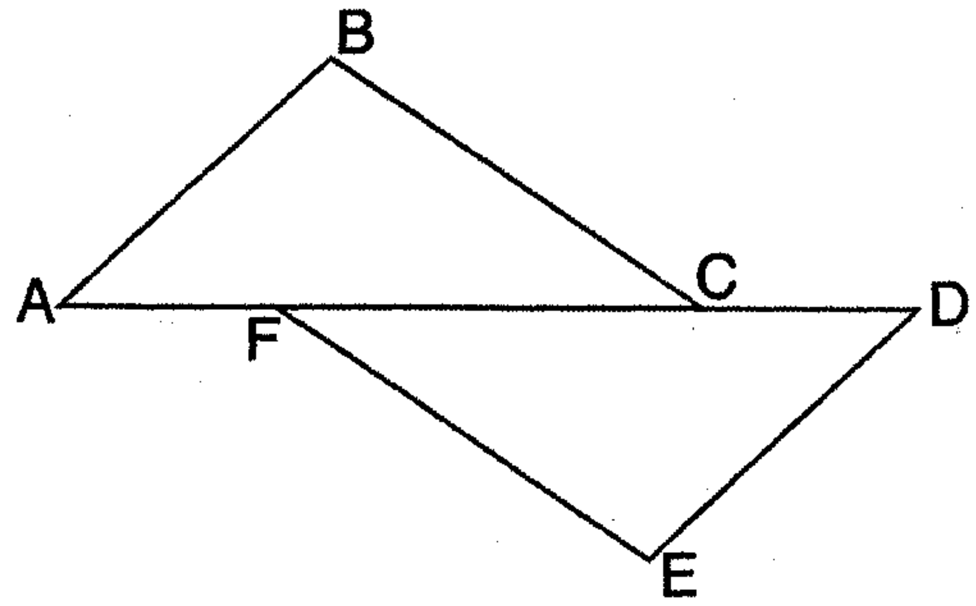
$(4t-5)(4t+3)=0$

$t=\frac{5}{4}=1.25$ negative solution



29 Complete the partial proof below for the accompanying diagram by providing reasons for steps 3, 6, 8, and 9.

Given: $\overline{AFCD}$
$\overline{AB} \perp \overline{BC}$
$\overline{DE} \perp \overline{EF}$
$\overline{BC} \parallel \overline{FE}$
$\overline{AB} \cong \overline{DE}$



Prove: $\overline{AC} \cong \overline{FD}$

Statements	Reasons
1 $\overline{AFCD}$	1 Given
2 $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$	2 Given
3 $\angle B$ and $\angle E$ are right angles.	3 Perpendicular line segments Form right angles
4 $\angle B \cong \angle E$	4 All right angles are congruent.
5 $\overline{BC} \parallel \overline{FE}$	5 Given
6 $\angle BCA \cong \angle EFD$	6 If two parallel lines are cut by a transversal, the alternate interior angles are congruent
7 $\overline{AB} \cong \overline{DE}$	7 Given
8 $\triangle ABC \cong \triangle DEF$	8 AAS
9 $\overline{AC} \cong \overline{FD}$	9 CPCTC

30 Solve for x: $\log_4 (x^2 + 3x) - \log_4 (x + 5) = 1$

$$\log_4\left(\frac{x^2+3x}{x+5}\right) = 1$$

$$\frac{x^2+3x}{x+5} = \frac{4^1}{1}$$

$$x^2+3x = 4(x+5)$$

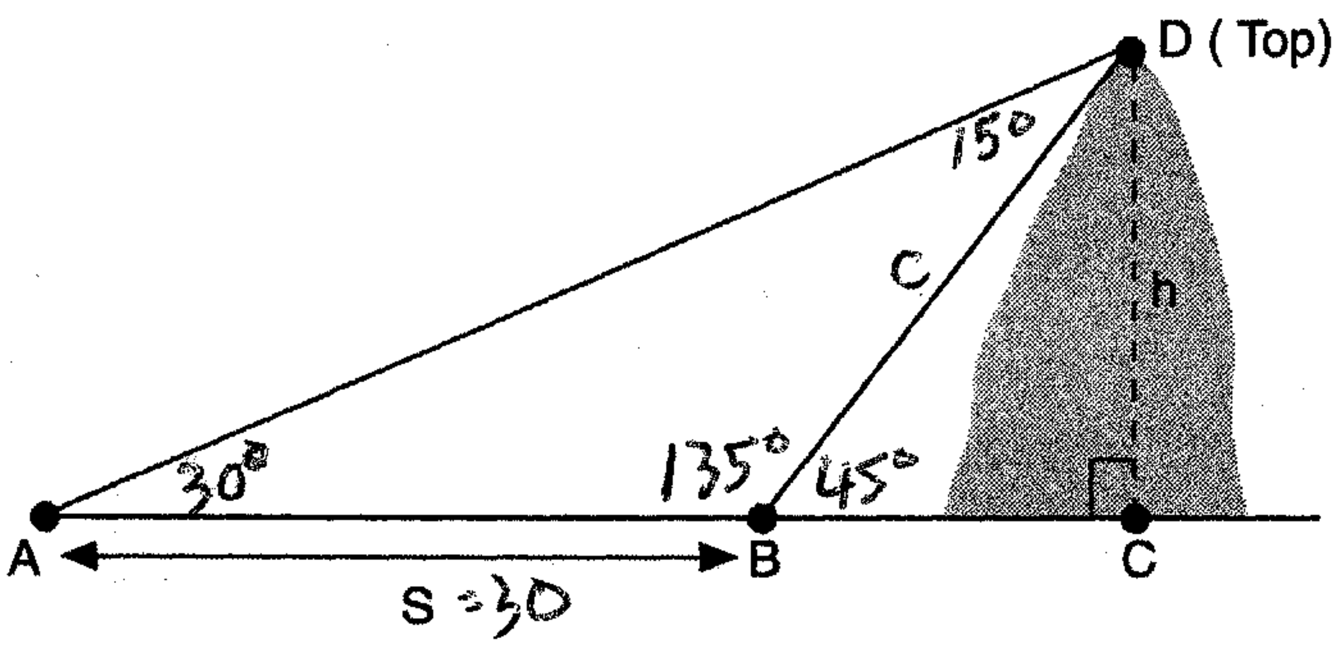
$$x^2+3x = 4x+20$$

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$5 \quad -4$$

31 A ship at sea heads directly toward a cliff on the shoreline. The accompanying diagram shows the top of the cliff, D, sighted from two locations, A and B, separated by distance S. If $m\angle DAC = 30$, $m\angle DBC = 45$, and $S = 30$ feet, what is the height of the cliff, to the *nearest foot*?



$$\frac{c}{\sin 30} = \frac{30}{\sin 15}$$

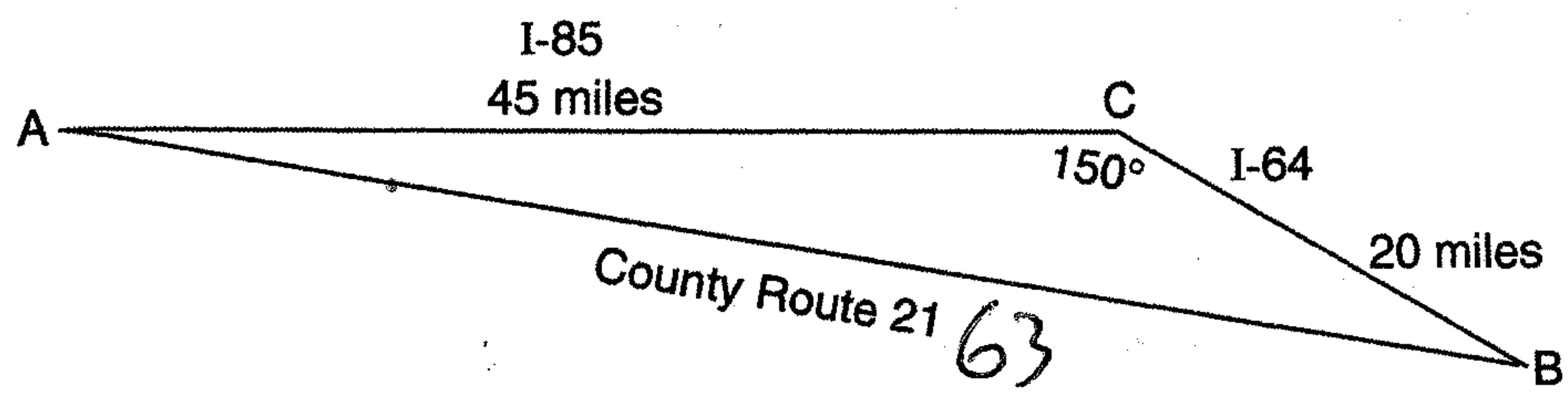
$$c = \frac{30 \sin 30}{\sin 15}$$

$$c \approx 58$$

$$\sin 45 = \frac{h}{58}$$

$$h \approx 41$$

32 Kieran is traveling from city A to city B. As the accompanying map indicates, Kieran could drive directly from A to B along County Route 21 at an average speed of 55 miles per hour or travel on the interstates, 45 miles along I-85 and 20 miles along I-64. The two interstates intersect at an angle of 150° at C and have a speed limit of 65 miles per hour. How much time will Kieran save by traveling along the interstates at an average speed of 65 miles per hour?



$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 20^2 + 45^2 - 2(20)(45) \cos 150$

$c^2 = 3984$

$c \approx 63$

If kieran drives directly from A to B, the trip will take 1.15 hours $\frac{63 \text{ miles}}{55 \text{ mph}}$

If kieran drives from A to B along the interstates, the trip will take 1 hour $\frac{45+20}{65}$

kieran will save .15 hours, or 9 minutes

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 On a monitor, the graphs of two impulses are recorded on the same screen, where $0° \le x < 360°$. The impulses are given by the following equations:

$$y = 2\sin^2 x$$
$$y = 1 - \sin x$$

Find all values of x, in degrees, for which the two impulses meet in the interval $0° \le x < 360°$. [Only an algebraic solution will be accepted.]

$2\sin^2 x = 1 - \sin x$

$2\sin^2 x + \sin x - 1 = 0$

let $\sin x = a$

$2a^2 + a - 1 = 0$

$(2a-1)(a+1) = 0$

$2a - 1 = 0 \qquad a + 1 = 0$

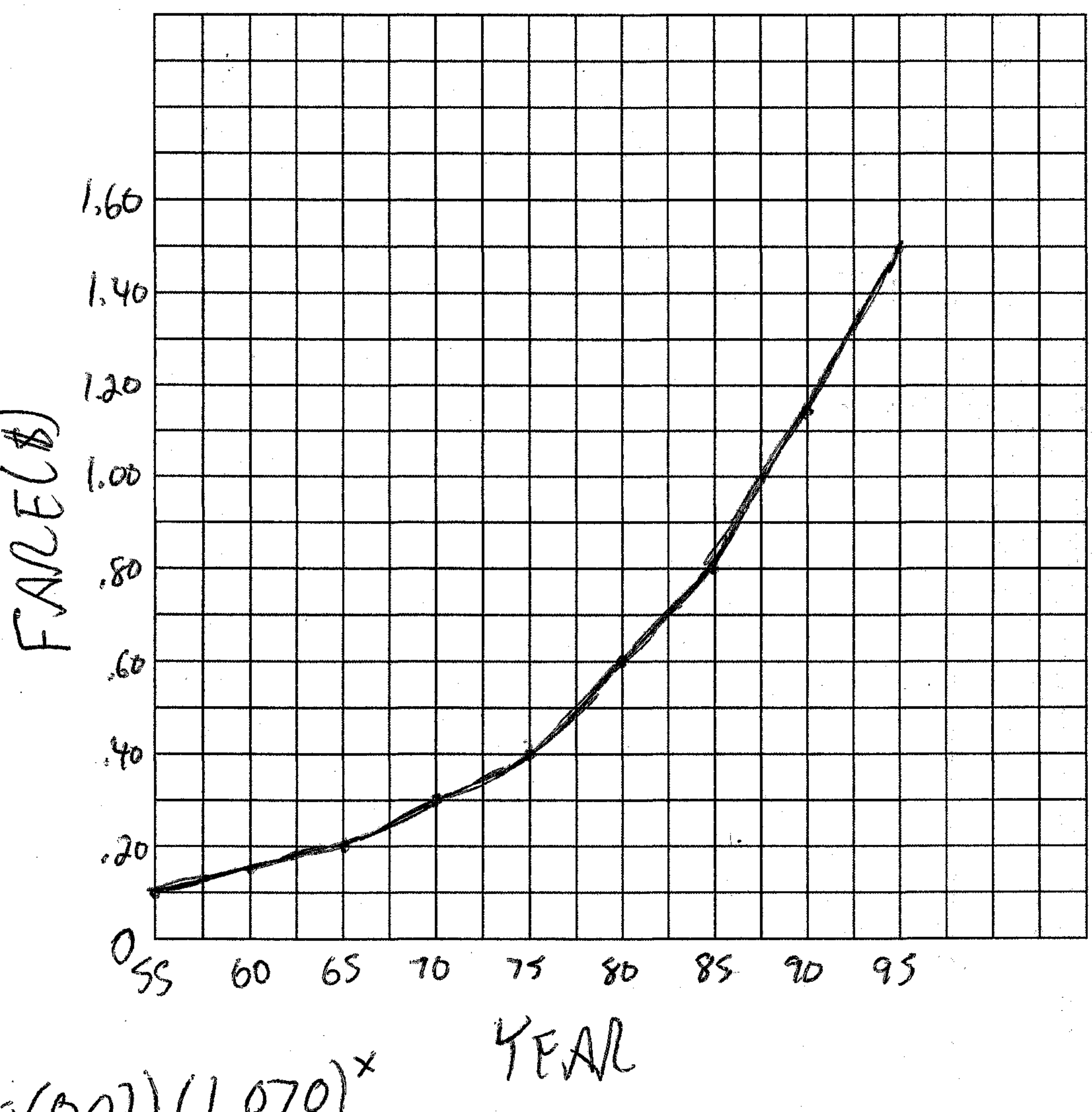
$a = \frac{1}{2} \qquad a = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = 30°, 150° \qquad x = 270°$

34 The table below, created in 1996, shows a history of transit fares from 1955 to 1995. On the accompanying grid, construct a scatter plot where the independent variable is years. State the exponential regression equation with the coefficient and base rounded to the *nearest thousandth*. Using this equation, determine the prediction that should have been made for the year 1998, to the *nearest cent*.

Year	55	60	65	70	75	80	85	90	95
Fare ($)	0.10	0.15	0.20	0.30	0.40	0.60	0.80	1.15	1.50



$y = (.002)(1.070)^x$

$y = (.002)(1.070)^{98} \approx \$1.52$