The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Tuesday, January 27, 2009 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

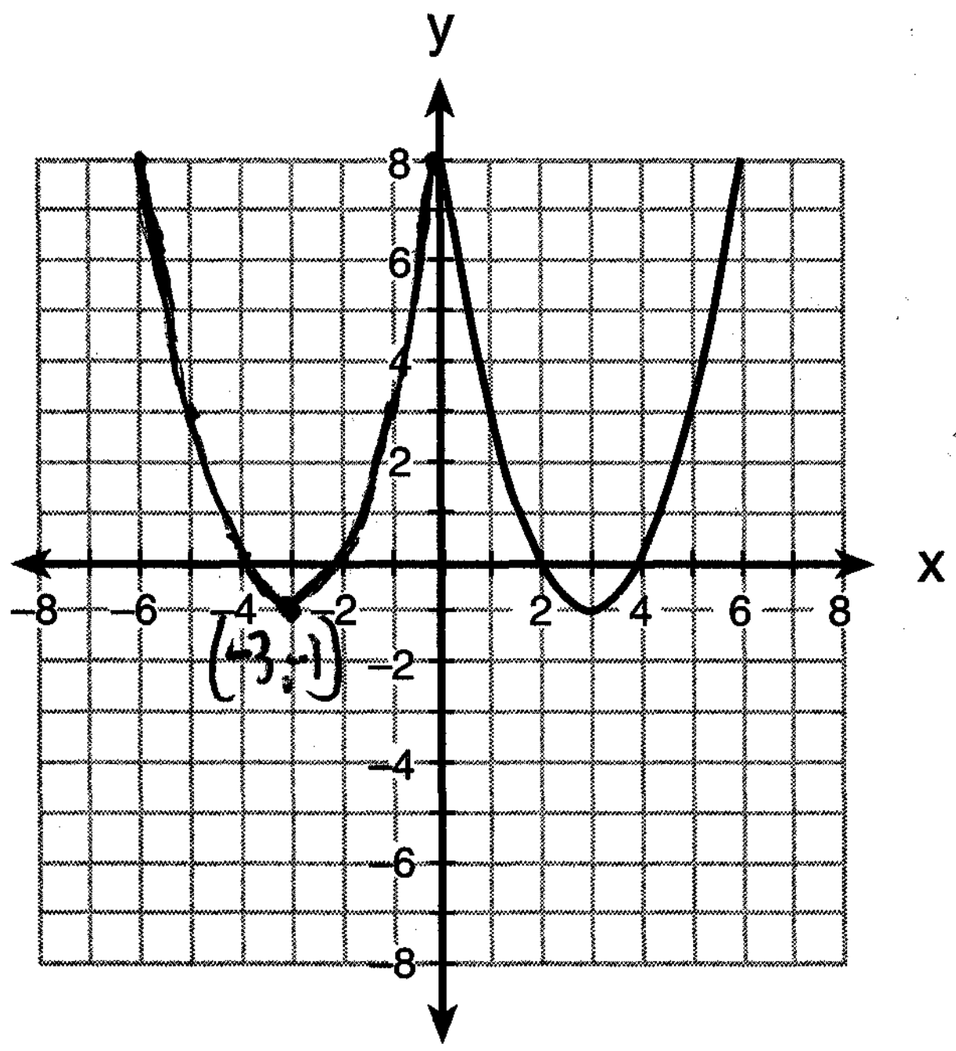
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

1 The parabola shown in the accompanying diagram undergoes a reflection in the y-axis.



What will be the coordinates of the turning point after the reflection?

(1) (3,–1)
(2) (3,1)
(3) (–3,1)
(4) (–3,–1)

2 The expression $\frac{5}{3+\sqrt{2}}$ is equivalent to

(1) $\frac{\sqrt{2}-15}{3}$
(2) $\frac{5\sqrt{2}-15}{5}$
(3) $\frac{15-5\sqrt{2}}{7}$
(4) $15-5\sqrt{2}$

$\frac{5}{3+\sqrt{2}} \cdot \left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{15-5\sqrt{2}}{9-2}$

$= \frac{15-5\sqrt{2}}{7}$

Use this space for computations.

3 If the probability that the Islanders will beat the Rangers in a game is $\frac{2}{5}$, which expression represents the probability that the Islanders will win *exactly* four out of seven games in a series against the Rangers?

(1) $\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^3$

(2) ${}_5C_2\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^3$

(3) ${}_7C_4\left(\frac{2}{5}\right)^4\left(\frac{2}{5}\right)^3$

(4) ${}_7C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^3$

$n = 7$
$r = 4$
$p = \frac{2}{5}$
$q = \frac{3}{5}$

$nC_r\, p^r\, q^{n-r}$

${}_7C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^3$

4 What is the solution of the inequality $x^2 - x - 6 < 0$?

(1) $-3 \le x \le -2$

(2) $-2 < x < 3$

(3) $1 < x < 6$

(4) $-3 < x < 2$

$x^2 - x - 6 < 0$
$(x-3)(x+2) < 0$
$x - 3 < 0$ and $x + 2 > 0$
$x < 3$ and $x > -2$

5 Which expression is equivalent to i^{55}?

(1) 1

(2) -1

(3) i

(4) $-i$

$\frac{55}{4} = 13$ R3 $\quad i^3 = -i$

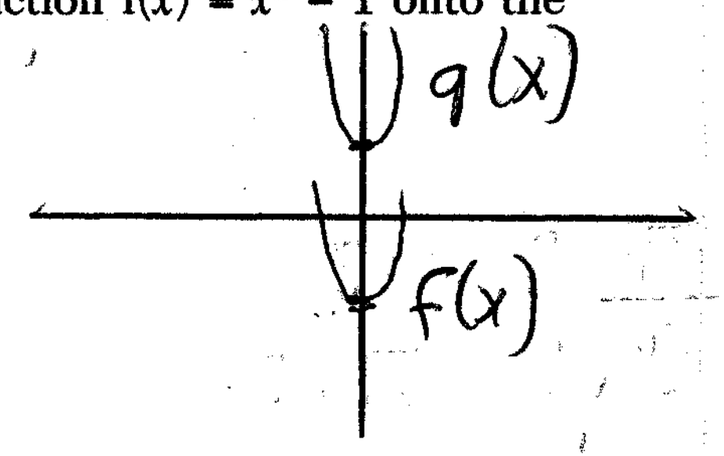
6 What is the translation that maps the function $f(x) = x^2 - 1$ onto the function $g(x) = x^2 + 1$?

(1) $T_{0,2}$

(2) $T_{0,1}$

(3) $T_{1,-1}$

(4) $T_{-1,1}$



7 The height of a swimmer's dive off a 10-foot platform into a diving pool is modeled by the equation $y = 2x^2 - 12x + 10$, where x represents the number of seconds since the swimmer left the diving board and y represents the number of feet above or below the water's surface. What is the farthest depth below the water's surface that the swimmer will reach?

(1) 6 feet

(2) 8 feet

(3) 10 feet

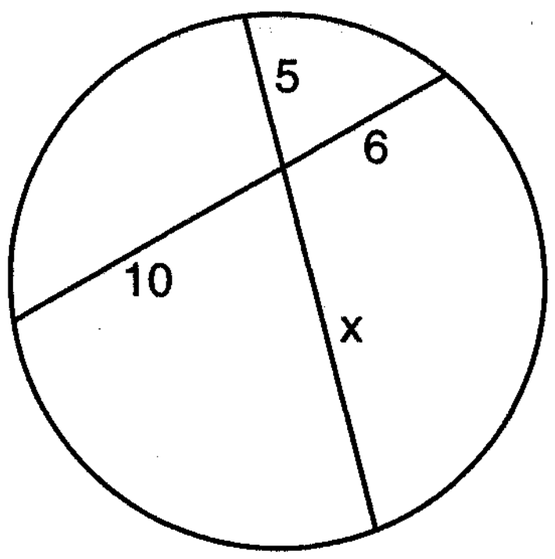
(4) 12 feet

Minimum occurs at
$x = \frac{-b}{2a} = \frac{12}{2(2)} = 3$

$y = 2(3)^2 - 12(3) + 10$
$= -8$

Use this space for computations.

8 The accompanying diagram shows two intersecting paths within a circular garden.



$5 \cdot x = 6 \cdot 10$

$\frac{5x}{5} = \frac{60}{5}$

$x = 12$

What is the length of the portion of the path marked x?

(1) $8\frac{1}{3}$

(2) 11

(3) 3

(4) 12

9 If $f(x) = 3x - 5$ and $g(x) = x - 9$, which expression is equivalent to $(f \circ g)(x)$?

(1) $4x - 14$

(2) $3x - 14$

(3) $3x - 32$

(4) $3x^2 - 32x + 45$

$f(g(x)) = 3(x-9)-5$

$3x-27-5$

$3x-32$

10 A central angle of a circular garden measures 2.5 radians and intercepts an arc of 20 feet. What is the radius of the garden?

(1) 8 ft

(2) 50 ft

(3) 100 ft

(4) 125 ft

$\theta = \frac{s}{r}$

$2.5 = \frac{20}{r}$

$r = 8$

11 What is a value of Arc sin $\left(-\frac{\sqrt{2}}{2}\right)$?

(1) $\frac{\pi}{4}$

(2) $-\frac{\pi}{4}$

(3) $\frac{\pi}{2}$

(4) $-\frac{\pi}{2}$

Use this space for computations.

12 A graphic designer is drawing a pattern of four concentric circles on the coordinate plane. The center of the circles is located at (–2,1). The smallest circle has a radius of 1 unit. If the radius of each of the circles is one unit greater than the largest circle within it, what would be the equation of the fourth circle?

r=4 $r^2=16$

(1) $(x-2)^2+(y+1)^2=4$
(2) $(x+2)^2+(y-1)^2=4$
(3) $(x-2)^2+(y+1)^2=16$
(4) $(x+2)^2+(y-1)^2=16$

13 Carol notices that the number of customers who visit her coffee shop varies inversely with the average daily temperature. Yesterday, the average temperature was 40° and she had 160 customers. If today's average temperature is 25°, how many customers should she expect?

$c_1 = c_2$

$\frac{160 \cdot 40}{25} = \frac{c \cdot 25}{25}$

$256 = c$

(1) 100
(2) 145
(3) 256
(4) 1,000

Function

14 Given the relation A: {(3,2), (5,3), (6,2), (7,4)}

Which statement is true?

A^{-1}: {(2,3), (3,5), (2,6), (4,7)}

Not a function

(1) Both A and A^{-1} are functions.
(2) Neither A nor A^{-1} is a function.
(3) Only A is a function.
(4) Only A^{-1} is a function.

15 The expression $\cot\theta \cdot \sec\theta$ is equivalent to

$\cot\theta \cdot \sec\theta$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$\frac{1}{\sin\theta}$

$\csc\theta$

(1) $\frac{\cos\theta}{\sin^2\theta}$
(2) $\frac{\sin\theta}{\cos^2\theta}$
(3) $\csc\theta$
(4) $\sin\theta$

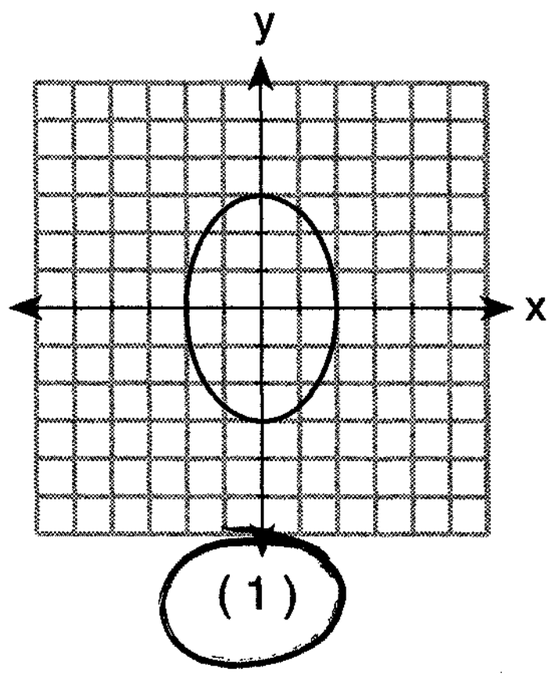
Use this space for computations.

16 If $z_1 = -3 + 2i$ and $z_2 = 4 - 3i$, in which quadrant does the graph of $(z_2 - z_1)$ lie?

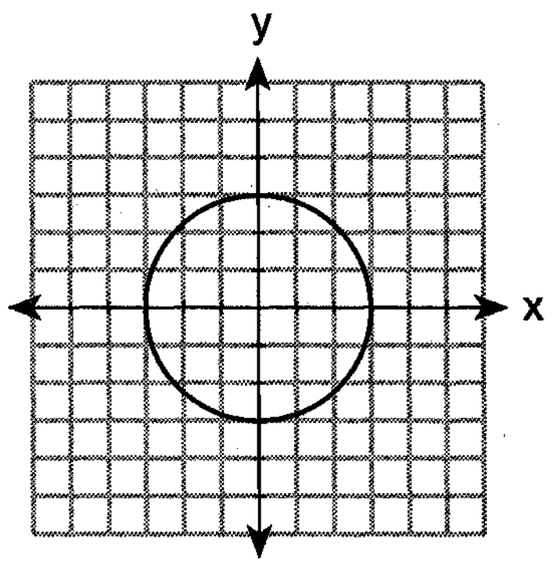
(1) I
(2) II
(3) III
(4) IV

$z_2 - z_1$
$4 - 3i - (-3 + 2i)$
$7 - 5i$

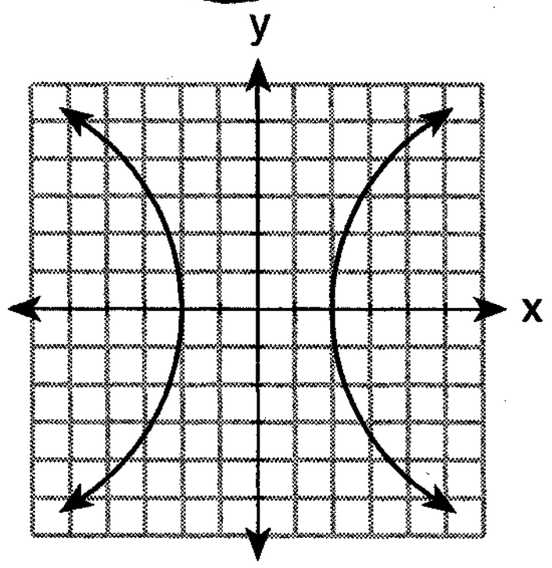
17 Which graph represents the equation $9x^2 = 36 - 4y^2$?



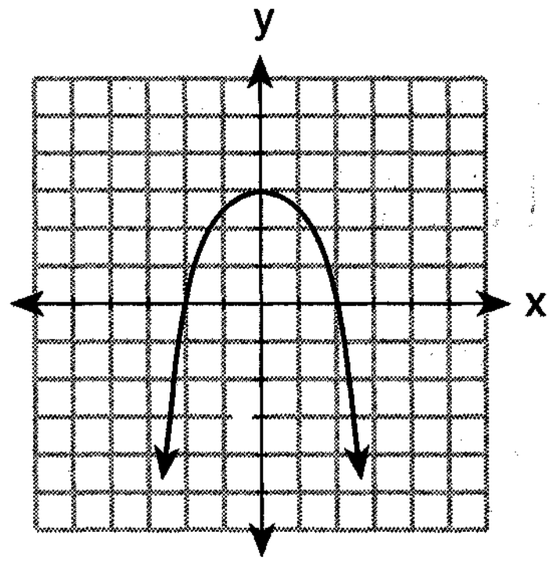
(1)



(3)



(2)



(4)

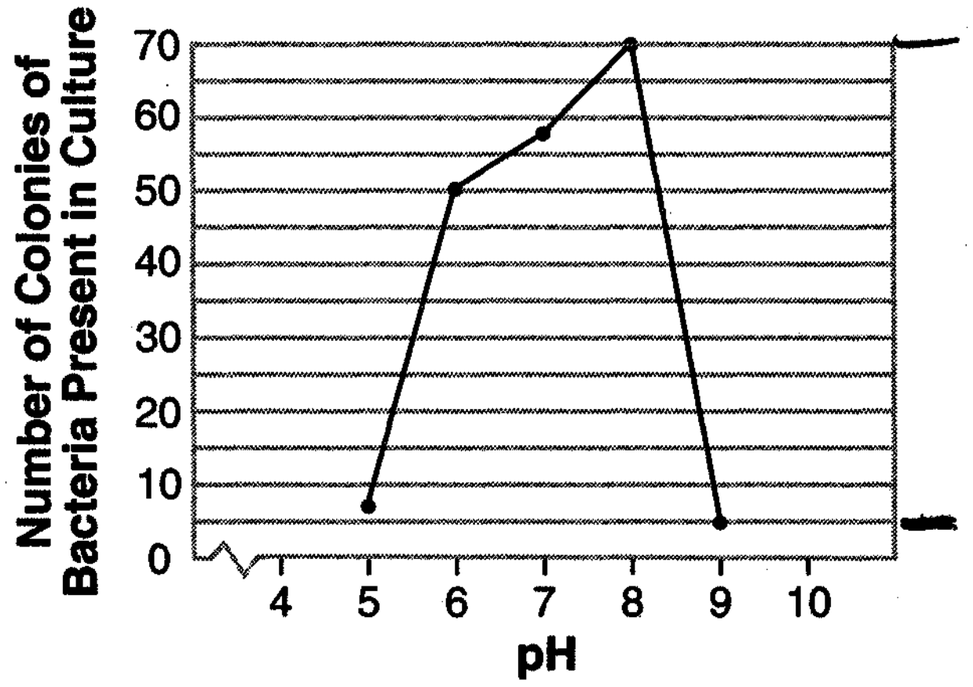
$9x^2 = 36 - 4y^2$
$+4y^2 \qquad +4y^2$

$\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

Use this space for computations.

18 The accompanying graph illustrates the presence of a certain strain of bacteria at various pH levels.



What is the range of this set of data?

(1) $5 \leq x \leq 9$
(2) $5 \leq x \leq 70$
(3) $0 \leq y \leq 70$
(4) $5 \leq y \leq 70$

19 Juan has been told to write a quadratic equation where the sum of the roots is equal to –3 and the product of the roots is equal to –9. Which equation meets these requirements?

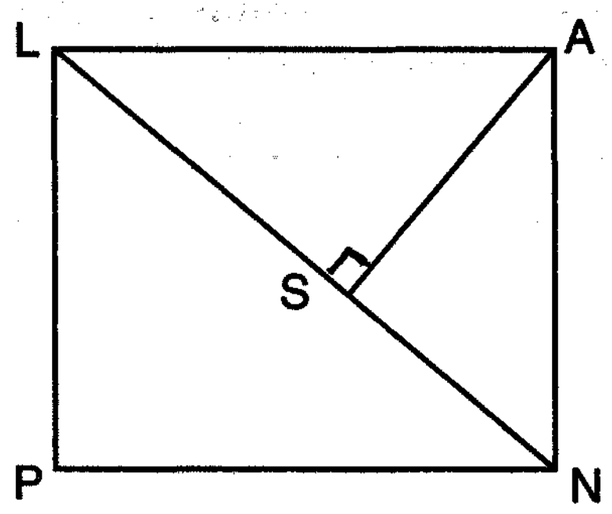
(1) $x^2 + 3x + 9 = 0$
(2) $x^2 - 12x + 27 = 0$
(3) $2x^2 + 6x - 18 = 0$
(4) $(x + 3)(x + 9) = 0$

sum of roots $\frac{-b}{a} = \frac{-6}{2} = -3$

product of roots $\frac{c}{a} = \frac{-18}{2} = -9$

Use this space for computations.

20 The accompanying diagram shows part of the architectural plans for a structural support of a building. *PLAN* is a rectangle and $\overline{AS} \perp \overline{LN}$.



Which equation can be used to find the length of $\overline{AS}$?

(1) $\frac{LS}{AS} = \frac{AS}{SN}$

(2) $\frac{AN}{LN} = \frac{AS}{LS}$

(3) $\frac{AS}{SN} = \frac{AS}{LS}$

(4) $\frac{AS}{LS} = \frac{LS}{SN}$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 Solve for x: $\sqrt{x+18} - 2 = 2$

$+2 \quad +2$

$\sqrt{x+18} = 4$

$x+18 = 16$

$-18 \quad -18$

$x = -2$

22 Evaluate: $\sum_{n=1}^{3}\left(\sin\frac{n\pi}{2}\right)$

n	$\sin\frac{n\pi}{2}$
1	$\sin\frac{1}{2}\pi = 1$
2	$\sin\frac{2}{2}\pi = 0$
3	$\sin\frac{3}{2}\pi = -1$

$1 + 0 + (-1) = 0$

23 Given a starting population of 100 bacteria, the formula $b = 100(2^t)$ can be used to find the number of bacteria, b, after t periods of time. If each period is 15 minutes long, how many minutes will it take for the population of bacteria to reach 51,200?

$$\frac{51,200}{100} = \frac{100 \cdot 2^t}{100}$$

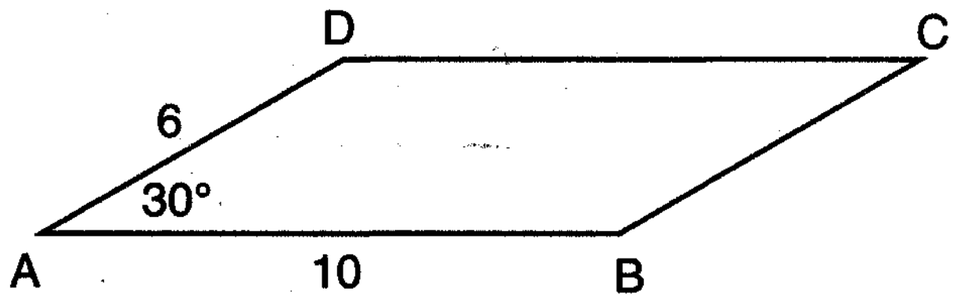
$$512 = 2^t$$

$$\log 512 = \log 2^t$$

$$\frac{\log 512}{\log 2} = \frac{t \log 2}{\log 2}$$

$9 = t$ $\qquad 15 \times 9 = 135$

24 In the accompanying diagram of parallelogram $ABCD$, $m\angle A = 30$, $AB = 10$, and $AD = 6$. What is the area of parallelogram $ABCD$?



$A = 6 \cdot 10 \sin 30 = 30$

25 What is the solution of the inequality $|2x - 5| \leq 11$?

$2x - 5 \leq 11$

$+5 \quad +5$

$\frac{2x}{2} \leq \frac{16}{2}$

$x \leq 8$

$2x - 5 \geq -11$

$+5 \quad +5$

$\frac{2x}{2} \geq \frac{-6}{2}$

$x \geq -3$

$-3 \leq x \leq 8$

26 The volume of Earth can be calculated by using the formula $V = \frac{4}{3}\pi r^3$. Solve for r in terms of V.

$\frac{3}{4\pi} \cdot V = \frac{4}{3}\pi r^3 \cdot \frac{3}{4\pi}$

$\frac{3V}{4\pi} = r^3$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 The average monthly high temperatures, in degrees Fahrenheit, for Binghamton, New York, are given below.

January	28	July	78
February	31	August	76
March	41	September	68
April	53	October	57
May	68	November	44
June	73	December	33

For these temperatures, find, to the *nearest tenth*, the mean, the population standard deviation, and the number of months that fall within one standard deviation of the mean.

$\bar{x} = 54.2$

$\sigma_x = 17.6$

$54.2 - 17.6$

36.6

$54.2 + 17.6$

71.8

6 months

28 Perform the indicated operations and express in simplest form:

$$\frac{3x^2 + 12x - 15}{x^2 + 2x - 15} \div \frac{3x^2 - 3x}{3x - x^2}$$

$$\frac{3(x^2+4x-5)}{(x+5)(x-3)} \times \frac{3x-x^2}{3x^2-3x}$$

$$\frac{\cancel{3}\cancel{(x+5)}\cancel{(x-1)}}{\cancel{(x+5)}(x-3)} \times \frac{\cancel{x}(3-x)}{\cancel{3}\cancel{x}\cancel{(x-1)}}$$

$$\frac{3-x}{x-3}$$

$$-1$$

29 In ΔABC, $a = 24$, $b = 36$, and $c = 30$. Find $m\angle A$ to the *nearest tenth of a degree*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$24^2 = 36^2 + 30^2 - 2(36)(30)\cos A$$

$$576 = 1296 + 900 - 2160\cos A$$

$$\frac{-1620}{-2160} = \frac{-2160\cos A}{-2160}$$

$$\frac{162}{216} = \cos A$$

$$\cos^{-1}\frac{162}{216} = A$$

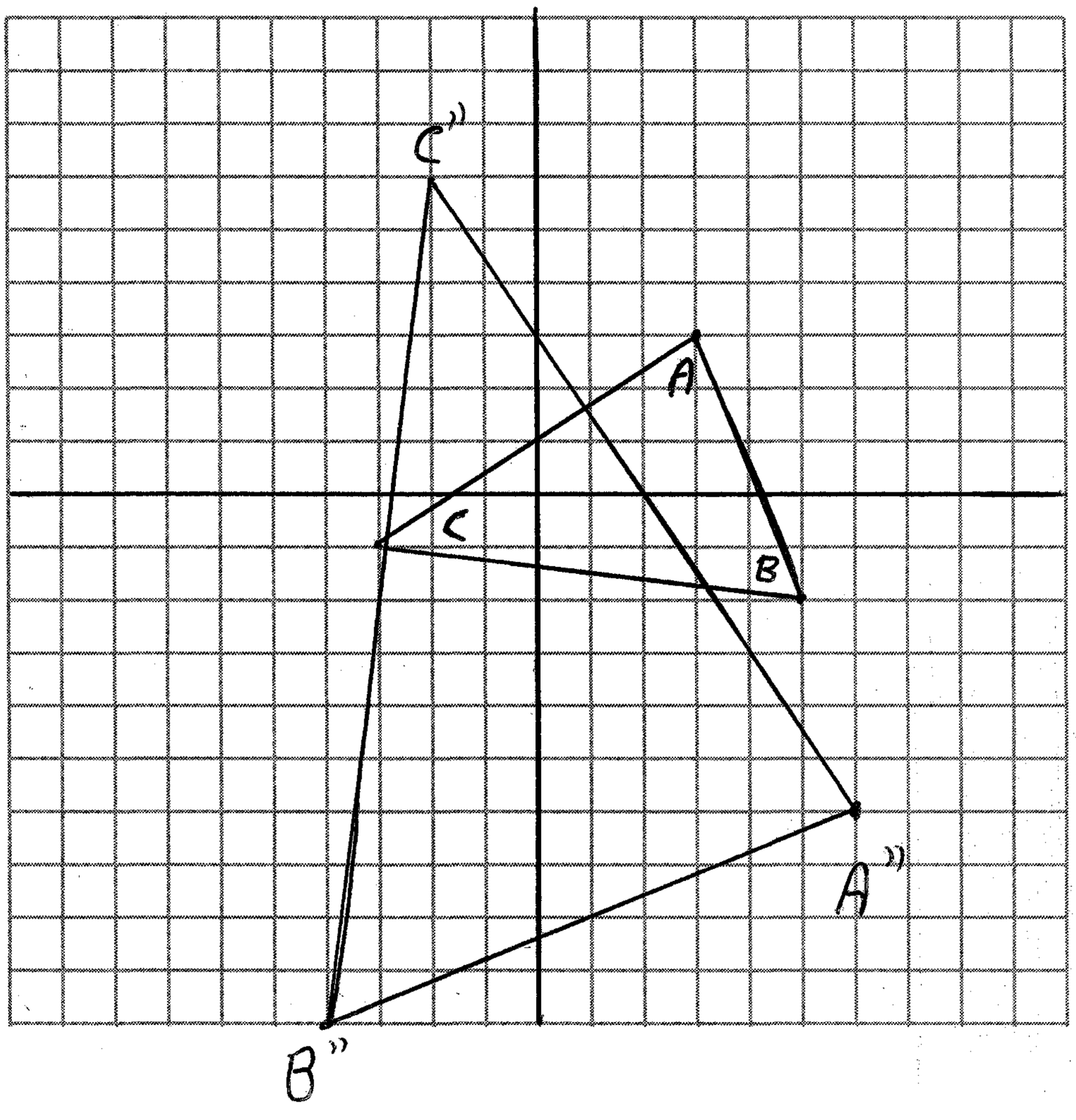
$$41.4 = A$$

30 Farmington, New York, has plans for a new triangular park. If plotted on a coordinate grid, the vertices would be $A(3,3)$, $B(5,-2)$, and $C(-3,-1)$. However, a tract of land has become available that would enable the planners to increase the size of the park, which is based on the following transformation of the original triangular park, $R_{270} \circ D_2$.

On the grid on the next page, graph and label both the original park ΔABC and its image, the new park $\Delta A''B''C''$, following the transformation.

ΔABC	$\Delta A'B'C'$	$\Delta A''B''C''$
A (3,3)	(6,6)	(6,-6)
B (5,-2)	(10,-4)	(-4,-10)
C (-3,-1)	(-6,-2)	(-2,6)

Question 30 continued



31 Find the roots of the equation $x^2 + 7 = 2x$ and express your answer in simplest $a + bi$ form.

$x^2 + 7 = 2x$

$-2x \quad -2x$

$x^2 - 2x + 7 = 0$

$a = 1$

$b = -2$

$c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-24}}{2}$$

$$= \frac{2 \pm \sqrt{4}\sqrt{6}\sqrt{-1}}{2}$$

$$= \frac{2 \pm 2i\sqrt{6}}{2}$$

$= 1 + i\sqrt{6}$ and $1 - i\sqrt{6}$

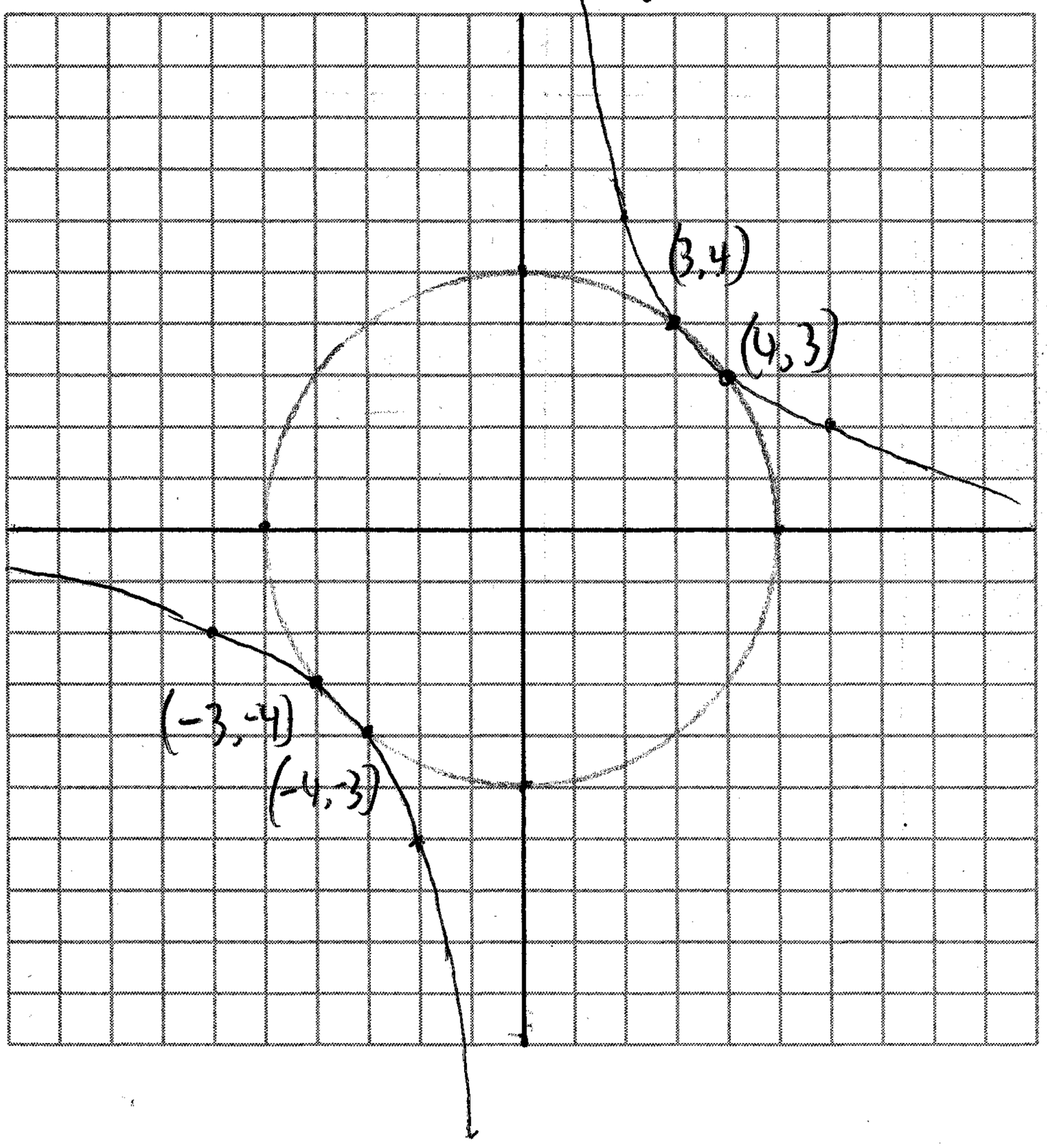
32 On the accompanying grid, graph the following system of equations over the interval $-6 \leq x \leq 6$.

$$x^2 + y^2 = 25$$
$$xy = 12$$

State the points of intersection.

$y = \frac{12}{x}$

x	y
-6	-2
-4	-3
-3	-4
-2	-6
-1	-12
1	12
2	6
3	4
4	3
6	2



Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 The accompanying table shows wind speed and the corresponding wind chill factor when the air temperature is 10°F.

Wind Speed (mi/h) x	**Wind Chill Factor** (°F) y
4	3
5	1
12	−5
16	−7
22	−10
31	−12

Write the logarithmic regression equation for this set of data, rounding coefficients to the *nearest ten thousandth*. $y = 13.0134 - 7.3135 \ln x$

Using this equation, find the wind chill factor, to the *nearest degree*, when the wind speed is 50 miles per hour. $13.0134 - 7.3135 \ln 50 \cong -16$

Based on your equation, if the wind chill factor is 0°, what is the wind speed, to the *nearest mile per hour*?

$0 = 13.0134 - 7.3135 \ln x$

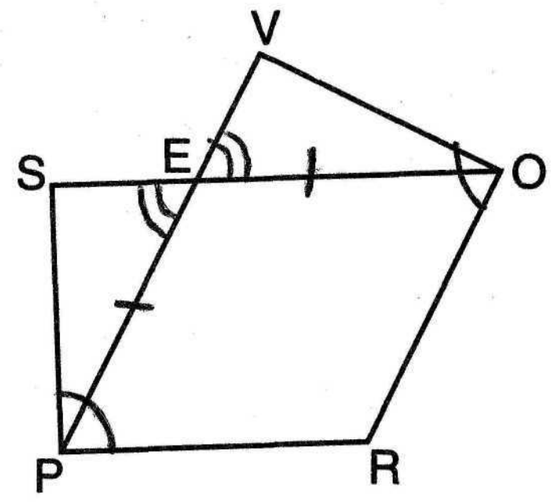
$-13.0134 = -13.0134$

$\frac{-13.0134}{-7.3135} = \frac{-7.3135 \ln x}{-7.3135}$

$\frac{13.0134}{7.3135} = \ln x$

$x = e^{\left(\frac{13.0134}{7.3135}\right)} \approx 6$

34 Given: $PROE$ is a rhombus, $\overline{SEO}$, $\overline{PEV}$, $\angle SPR \cong \angle VOR$



Prove: $\overline{SE} \cong \overline{EV}$

STATEMENT	REASON
(1) PROE is a rhombus, $\overline{SEO}$, $\overline{PEV}$, $\angle SPR \cong \angle VOR$	(1) Given
(2) $\angle SEP \cong \angle VEO$	(2) Vertical Angles
(3) $\overline{PE} \cong \overline{OE}$	(3) All sides of a rhombus are congruent
(4) $\angle EPR \cong \angle EOR$	(4) Opposite angles of a rhombus are congruent
(5) $\angle SPE \cong \angle VOE$	(5) Angle Subtraction Theorem
(6) $\triangle SPE \cong \triangle VOE$	(6) ASA
(7) $\overline{SE} \cong \overline{EV}$	(7) CPCTC