The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Friday, January 26, 2007 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: *Steve Sibol*

Print Your School's Name: *HS for Civil Rights*

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.
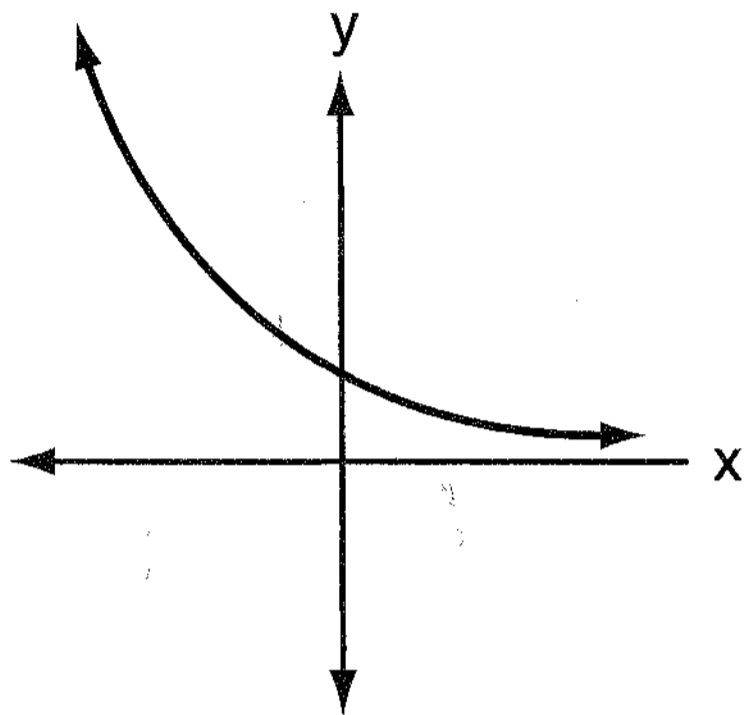
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.   [40]

1 Which equation best represents the accompanying graph?
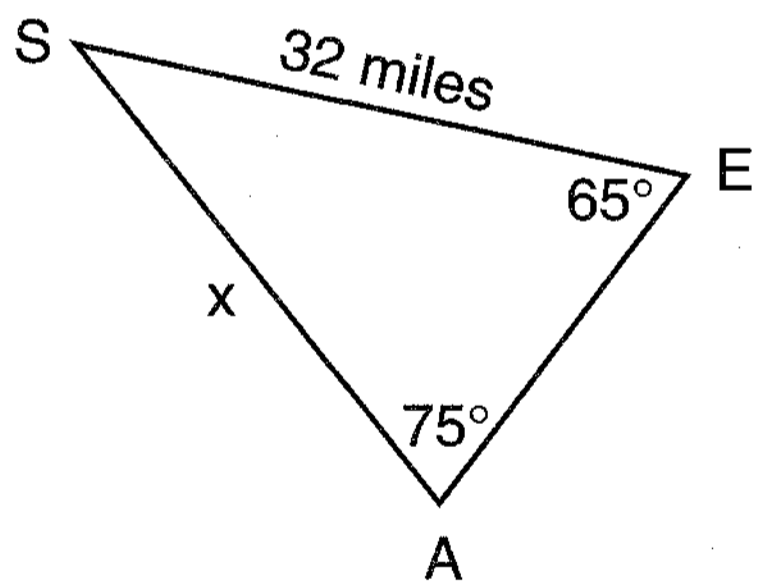

(1) $y = 2^x$

(2) $y = x^2 + 2$

(3) $y = 2^{-x}$

(4) $y = -2^x$

2 The accompanying diagram shows the approximate linear distances traveled by a sailboat during a race. The sailboat started at point S, traveled to points E and A, respectively, and ended at point S.

Based on the measures shown in the diagram, which equation can be used to find x, the distance from point A to point S?

(1) $\dfrac{x}{\sin 65°} = \dfrac{\sin 75°}{32}$

(2) $\dfrac{\sin 65°}{x} = \dfrac{\sin 75°}{32}$

(3) $\dfrac{x}{65} = \dfrac{32}{75}$

(4) $\dfrac{65}{x} = \dfrac{32}{75}$

3 If $\sqrt{x-a} = b$, $x > a$, which expression is equivalent to x?

(1) $b^2 - a$ (3) $b - a$

(2) $b^2 + a$ (4) $b + a$

$\sqrt{x-a} = b$

$x - a = b^2$
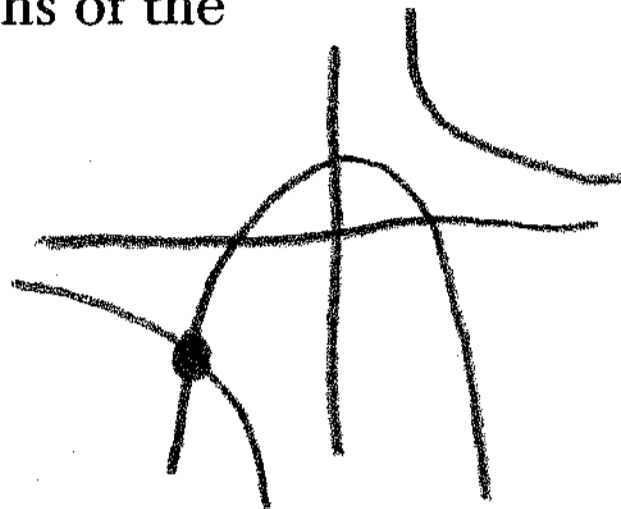
$x = b^2 + a$

4 What is the total number of points of intersection of the graphs of the equations $xy = 12$ and $y = -x^2 + 3$?

(1) 1   (3) 3

(2) 2   (4) 4

5 The expression i^{25} is equivalent to

(1) 1   (3) i

(2) -1 (4) $-i$

6 The expression $\dfrac{\frac{1}{3} + \frac{1}{3x}}{\frac{1}{x} + \frac{1}{3}}$ is equivalent to

(1) $\dfrac{x+1}{x+3}$ (3) $\dfrac{3x+3}{x+3}$

(2) 2   (4) $\frac{1}{3}$

$\dfrac{\frac{3x+3}{9x}}{\frac{3+x}{3x}} = \dfrac{\frac{3(x+1)}{9}}{\frac{x+3}{3}} = \dfrac{x+1}{x+3}$

7 The term "snowstorms of note" applies to all snowfalls over 6 inches. The snowfall amounts for snowstorms of note in Utica, New York, over a four-year period are as follows: 7.1, 9.2, 8.0, 6.1, 14.4, 8.5, 6.1, 6.8, 7.7, 21.5, 6.7, 9.0, 8.4, 7.0, 11.5, 14.1, 9.5, 8.6

What are the mean and population standard deviation for these data, to the *nearest hundredth*?

(1) mean = 9.46; standard deviation = 3.74

(2) mean = 9.46; standard deviation = 3.85

(3) mean = 9.45; standard deviation = 3.74

(4) mean = 9.45; standard deviation = 3.85

8 The expression $\dfrac{4}{5-\sqrt{13}}$ is equivalent to

(1) $\dfrac{5+\sqrt{13}}{3}$

(3) $\dfrac{2(5+\sqrt{13})}{19}$

(2) $\dfrac{5-\sqrt{13}}{3}$

(4) $\dfrac{2(5-\sqrt{13})}{19}$

$$\frac{4}{5-\sqrt{13}} \qquad \frac{5+\sqrt{13}}{5+\sqrt{13}}$$

$$\frac{20+4\sqrt{13}}{25-13}$$

$$\frac{\cancel{4}(5+\sqrt{13})}{\cancel{12}\,3}$$

$$\frac{5+\sqrt{13}}{3}$$

9 What is the value of b in the equation $4^{2b-3}=8^{1-b}$?

(1) $\dfrac{-3}{7}$

(3) $\dfrac{9}{7}$

(2) $\dfrac{7}{9}$

(4) $\dfrac{10}{7}$

$$\left(2^2\right)^{2b-3}=\left(2^3\right)^{1-b}$$

$$2^{4b-6}=2^{3-3b}$$

$$4b-6=3-3b$$
$$7b=9$$
$$b=\frac{9}{7}$$

10 What is the solution set of the inequality $|2x-1|<9$?

(1) $\{x\,|\,-4<x<5\}$

(3) $\{x\,|\,x<5\}$

(2) $\{x\,|\,x<-4 \text{ or } x>5\}$

(4) $\{x\,|\,x<-4\}$

$$2x-1<9 \text{ and } 2x-1>-9$$
$$2x<10 \text{ and } 2x>-8$$
$$x<5 \text{ and } x>-4$$

11 Which transformation could be used to make the graph of the equation $y=\sin x$ coincide with the graph of the equation $y=\cos x$?
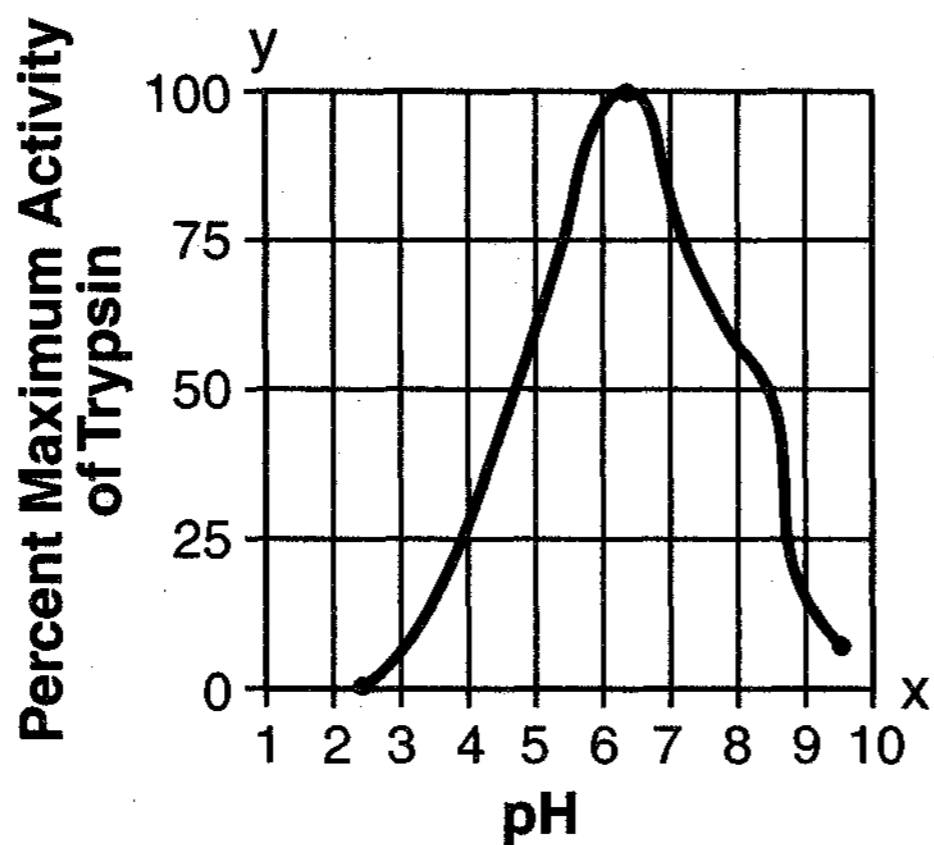
(1) translation

(3) dilation

(2) rotation

(4) point reflection

12 Data collected during an experiment are shown in the accompanying graph.

What is the range of this set of data?

(1) $2.5 \leq y \leq 9.5$ (3) $0 \leq y \leq 100$

(2) $2.5 \leq x \leq 9.5$ (4) $1 \leq x \leq 10$

13 Which is a true statement about the graph of the equation $y = x^2 - 7x - 60$?

(1) It is tangent to the x-axis.

(2) It does not intersect the x-axis.

(3) It intersects the x-axis in two distinct points that have irrational coordinates.

(4) It intersects the x-axis in two distinct points that have rational coordinates.

$b^2 - 4ac$

$a = 1 \quad b = -7 \quad c = -60$

$(-7)^2 - 4(1)(-60)$

$49 + 240$

$289 > 0$

$\sqrt{289} = 17$

14 Which quadratic equation has the roots $3 + i$ and $3 - i$?

(1) $x^2 + 6x - 10 = 0$ (3) $x^2 - 6x + 10 = 0$

(2) $x^2 + 6x + 8 = 0$ (4) $x^2 - 6x - 8 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(1)(10)$

$36 - 40$

$-4 < 0$

$a = 1$
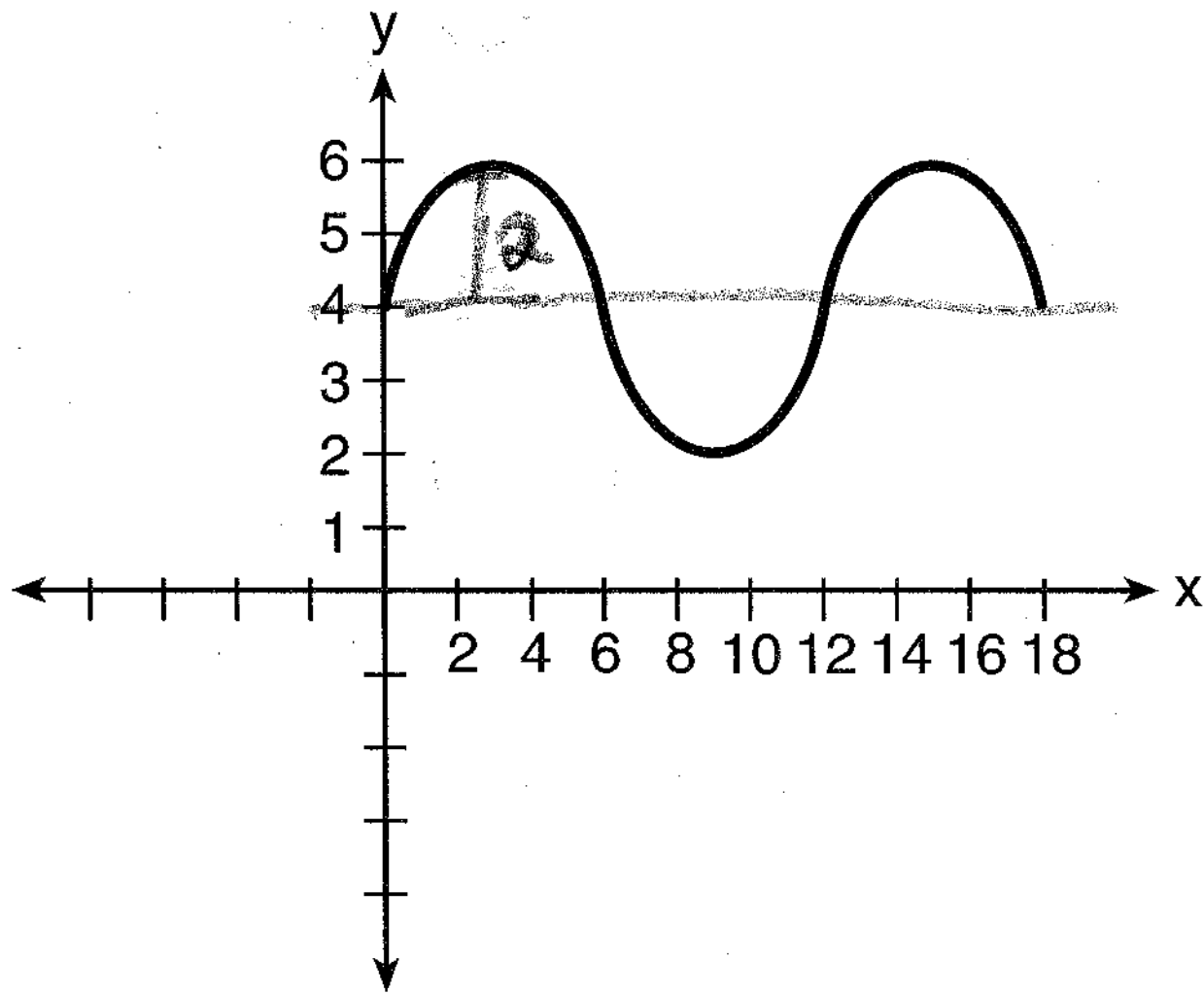
$b = -6$

$c = 10$

The sum of the roots equals $\frac{-b}{a}$ $3 + i + 3 - i = 6 = \frac{-b}{a}$

The product of the roots equals $\frac{c}{a}$ $(3+i)(3-i) = 9 - i^2$

$9 - (-1) = 10 = \frac{c}{a}$

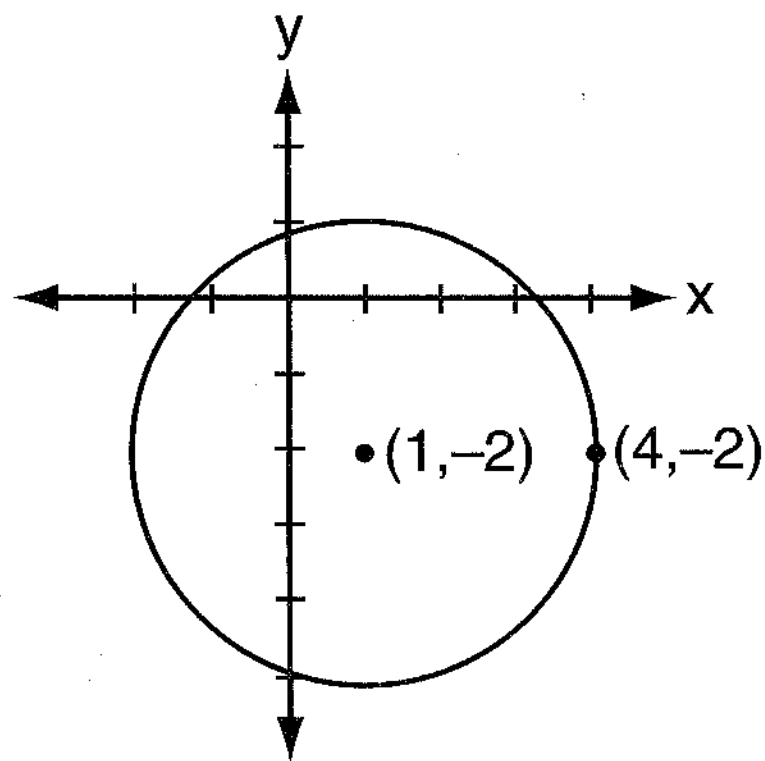therefore the roots of $x^2 - 6x + 10 = 0$ are imaginary.

15 What is the amplitude of the function shown in the accompanying graph?

(1) 1.5

(2) 2

(3) 6

(4) 12

16 Which equation represents the circle shown in the accompanying graph?

(1) $(x - 1)^2 - (y + 2)^2 = 9$

(2) $(x - 1)^2 + (y + 2)^2 = 9$

(3) $(x + 1)^2 - (y - 2)^2 = 9$

(4) $(x + 1)^2 + (y - 2)^2 = 9$

17 A black hole is a region in space where objects seem to disappear. A formula used in the study of black holes is the Schwarzschild formula,

$$R = \frac{2GM}{c^2}.$$

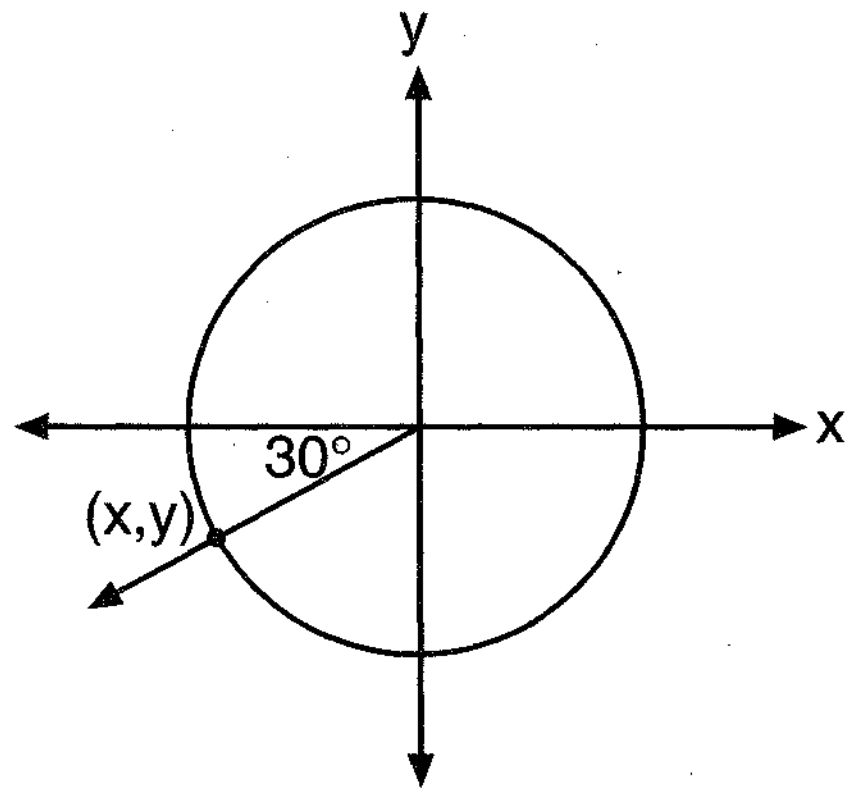Based on the laws of logarithms, log R can be represented by

(1) $2 \log G + \log M - \log 2c$
(2) $\log 2G + \log M - \log 2c$
(3) $\log 2 + \log G + \log M - 2 \log c$
(4) $2 \log GM - 2 \log c$

Handwritten work:

$\log \frac{2GM}{c^2}$

$\log 2 + \log G + \log M - \log c^2$

$\log 2 + \log G + \log M - 2\log c$

18 In the unit circle shown in the accompanying diagram, what are the coordinates of (x,y)?

(1) $\left(-\frac{\sqrt{3}}{2}, -0.5\right)$

(2) $\left(-0.5, -\frac{\sqrt{3}}{2}\right)$

(3) $(-30, -210)$

(4) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

Handwritten work:

$\cos 210° = -.866 = -\frac{\sqrt{3}}{2}$

$\sin 210° = -\frac{1}{2}$

19 Which transformation represents a dilation?

(1) $(8,4) \rightarrow (11,7)$ (3) $(8,4) \rightarrow (-4,-8)$

(2) $(8,4) \rightarrow (-8,4)$ (4) $(8,4) \rightarrow (4,2)$

20 In $\triangle ABC$, $m\angle A = 30$, $a = 14$, and $b = 20$. Which type of angle is $\angle B$?

(1) It must be an acute angle.

(2) It must be a right angle.

(3) It must be an obtuse angle.

(4) It may be either an acute angle or an obtuse angle.
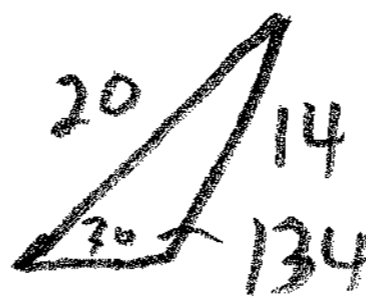
$$\frac{14}{\sin 30} = \frac{20}{\sin x}$$

$$14 \sin x = 20 \sin 30$$

$$\sin x = \frac{20 \sin 30}{14}$$

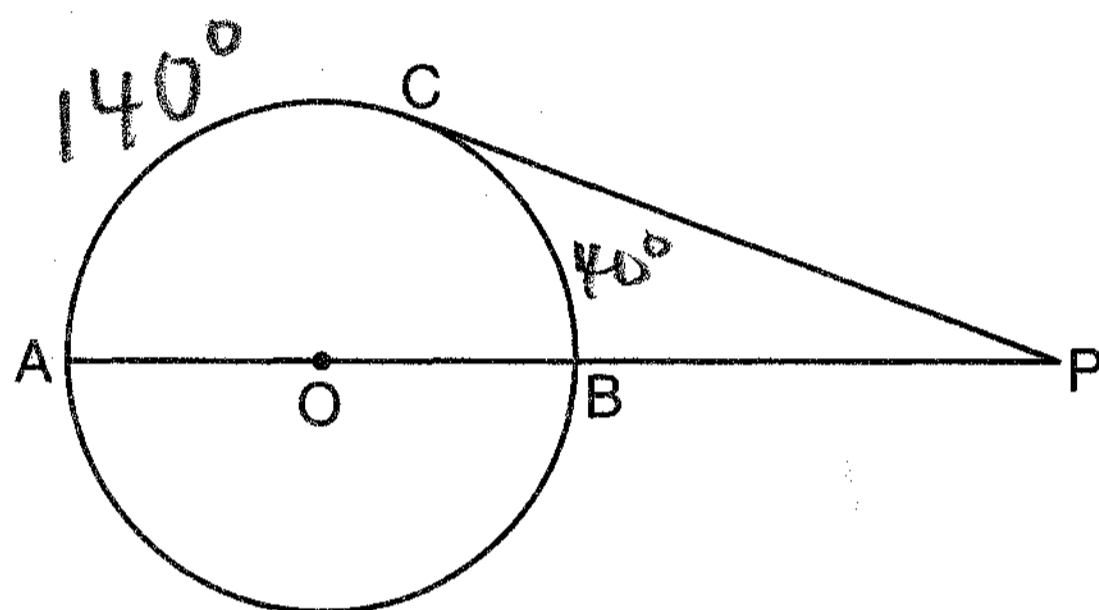$$\sin x = .714$$

$$x = 46°$$

$$30 + 46 < 180$$

$$180 - 46 = 134°$$

$$134 + 30 < 180$$

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 In the accompanying diagram of circle O, diameter $\overline{AOB}$ is extended through B to external point P, tangent $\overline{PC}$ is drawn to point C on the circle, and $m\widehat{AC}:m\widehat{BC}=7:2$. Find $m\angle CPA$.

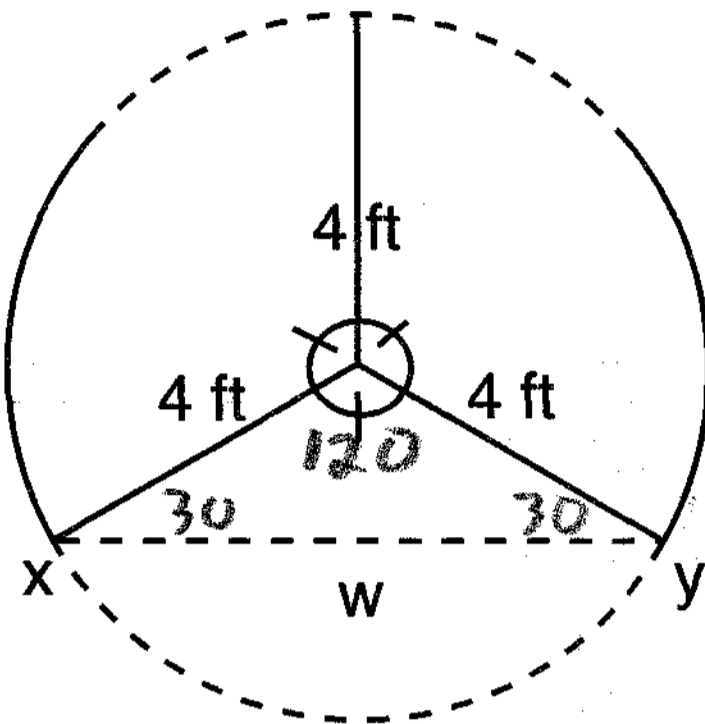(Not drawn to scale)

$$m\,\widehat{AC} = \frac{7}{9} \times 180 = 140$$

$$m\,\widehat{BC} = \frac{2}{9} \times 180 = 40$$
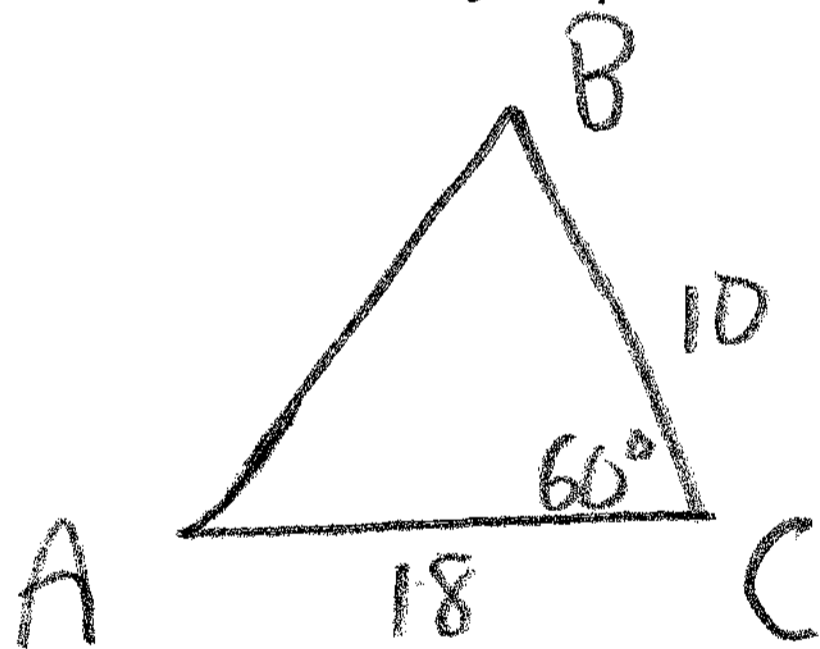
$$m\angle CPA = \frac{140-40}{2} = 50°$$

22 The accompanying diagram shows a revolving door with three panels, each of which is 4 feet long. What is the width, w, of the opening between x and y, to the *nearest tenth of a foot?*

$$\frac{W}{\sin 120} = \frac{4}{\sin 30}$$

$$W = \frac{4 \sin 120}{\sin 30} = 6.9$$

23 In $\triangle ABC$, $AC = 18$, $BC = 10$, and $\cos C = \frac{1}{2}$. Find the area of $\triangle ABC$ to the *nearest tenth of a square unit.*
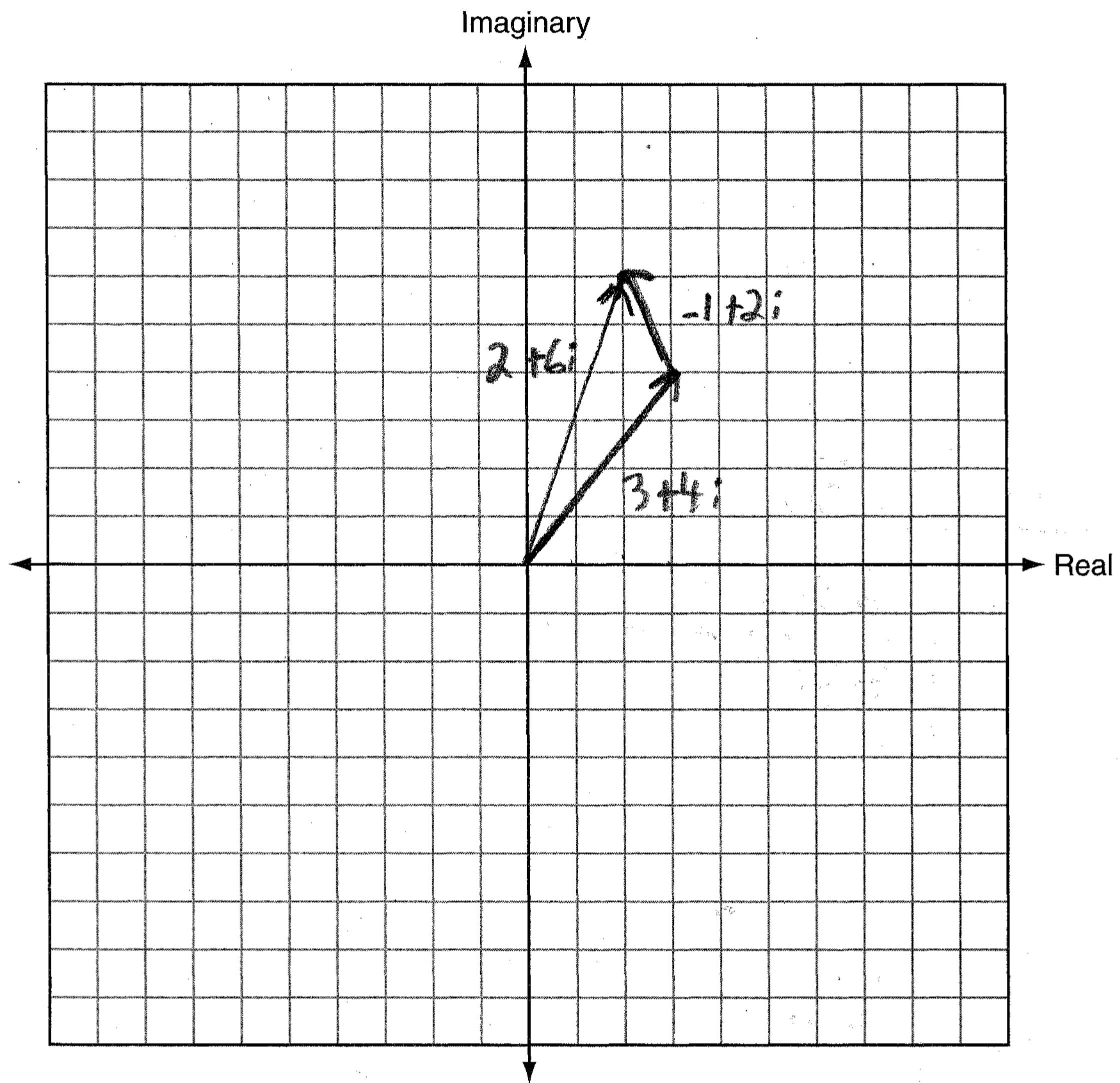

$$\cos C = \frac{1}{2}$$

$$C = \cos^{-1} \frac{1}{2}$$
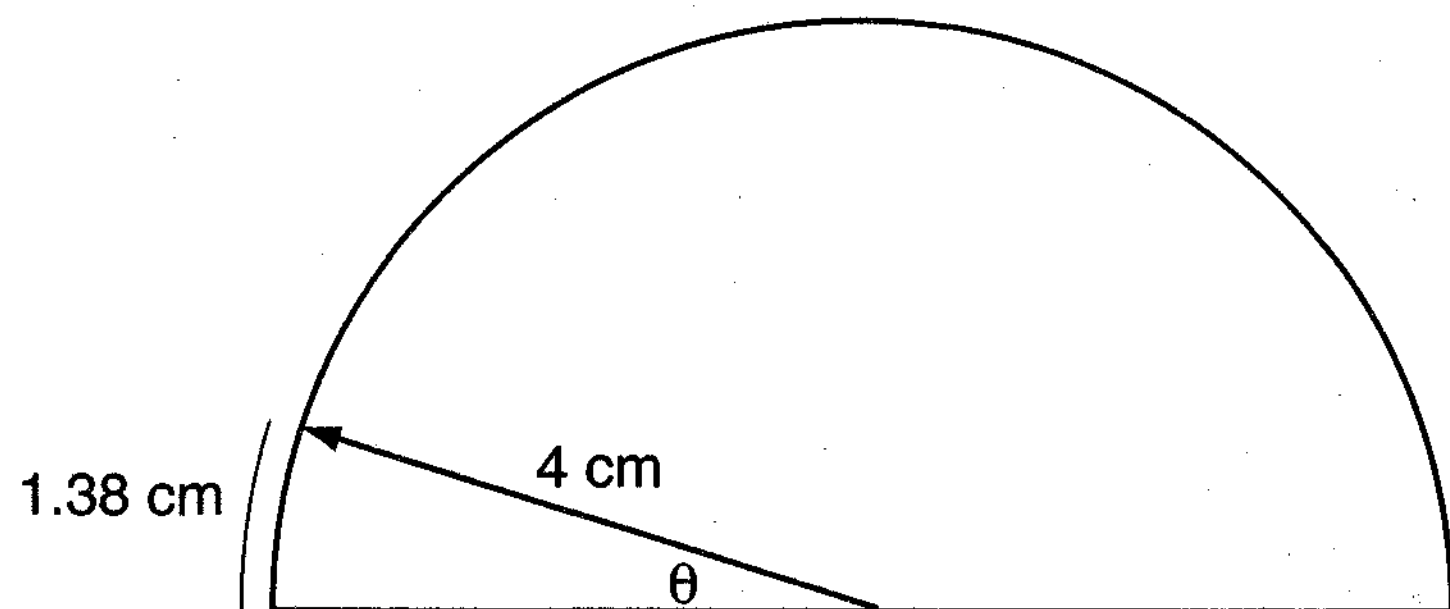
$$C = 60°$$

$$\text{Area} = \frac{1}{2} \, ab \sin C$$

$$= \frac{1}{2}(10)(18) \sin 60°$$

$$= 77.9$$

24 On the accompanying set of axes, graphically represent the sum of $3 + 4i$ and $-1 + 2i$.

Imaginary

2+6i

-1+2i

3+4i

Real

25 As shown in the accompanying diagram, a dial in the shape of a semicircle has a radius of 4 centimeters. Find the measure of θ, in radians, when the pointer rotates to form an arc whose length is 1.38 centimeters.

1.38 cm   4 cm   θ

$$\frac{1.38}{8\pi} = \frac{\theta}{2\pi}$$

$$\theta = \frac{1.38 \cdot 2\pi}{4 \; 8\pi} = .345$$

26 What is the fourth term in the expansion of $(2x - y)^5$?

$$_5C_3 (2x)^2 (-x)^3$$

$$10 \cdot 4x^2 \cdot -y^3$$

$$-40x^2 y^3$$

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 Find, to the *nearest degree*, all values of θ in the interval $0° \leq \theta \leq 180°$ that satisfy the equation $8 \cos^2 \theta - 2 \cos \theta - 1 = 0$.

Let $x = \cos \theta$

$$8x^2 - 2x - 1 = 0$$

$$(4x + 1)(2x - 1) = 0$$

$4x + 1 = 0 \qquad\qquad 2x - 1 = 0$

$x = -\frac{1}{4} \qquad\qquad x = \frac{1}{2}$

$\cos \theta = -\frac{1}{4} \qquad\qquad \cos \theta = \frac{1}{2}$

$\theta = \cos^{-1} -\frac{1}{4} \qquad\qquad \theta = \cos^{-1} \frac{1}{2}$

$\theta = 104° \qquad\qquad \theta = 60°$

28 Since January 1980, the population of the city of Brownville has grown according to the mathematical model $y = 720{,}500(1.022)^x$, where x is the number of years since January 1980.

Explain what the numbers 720,500 and 1.022 represent in this model.

If this trend continues, use this model to predict the year during which the population of Brownville will reach 1,548,800. [The use of the grid on the next page is optional.]

720,500 represents the population of Brownsville as of January, 1980

1.022 represents the growth factor. The population is growing at 2.2%.

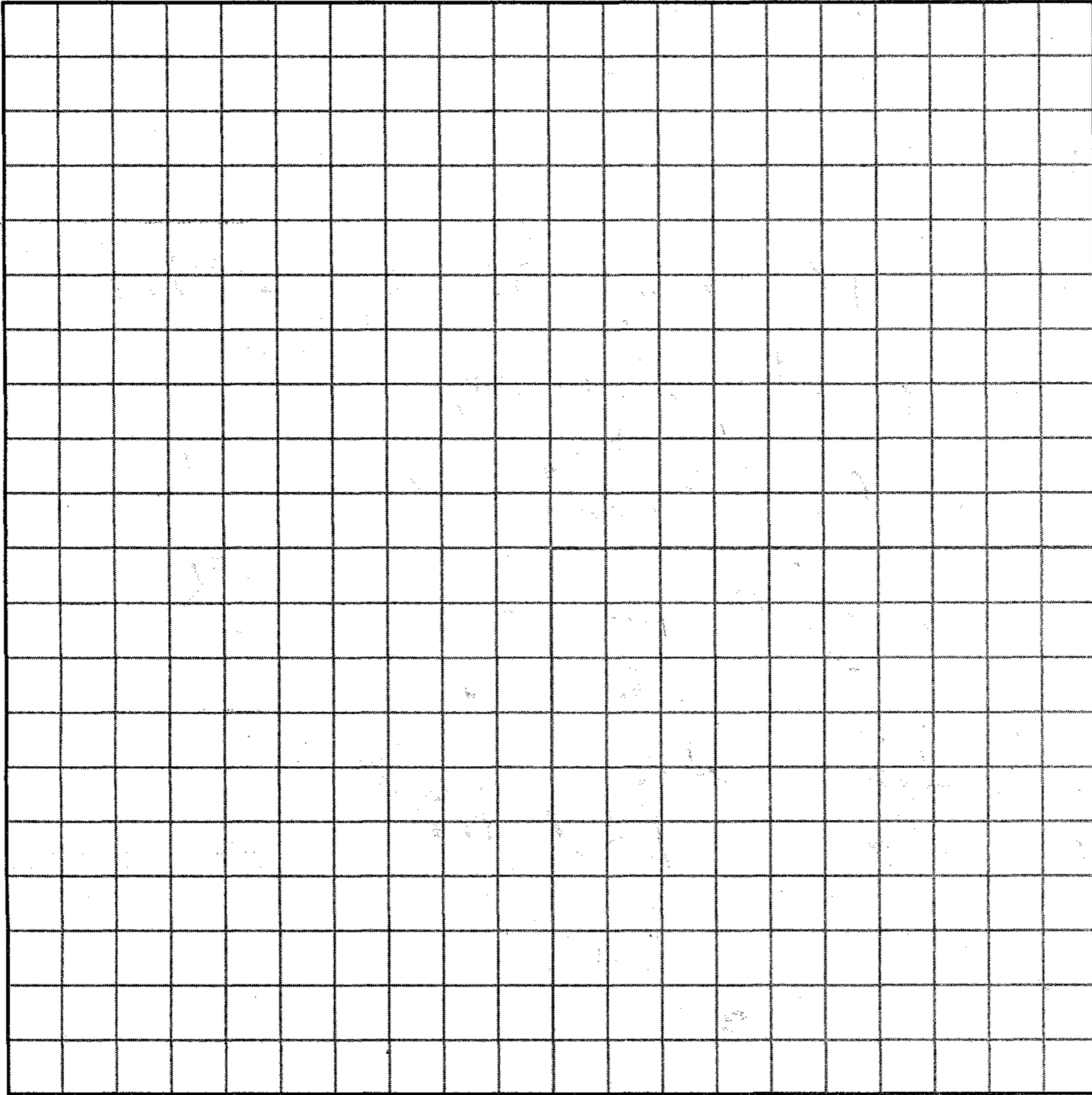$$\frac{1548800}{720{,}500} = \frac{720{,}500\,(1.022)^x}{720{,}500}$$

$$\log 2.1496 = \log 1.022^x$$

$$\log 2.1496 = x \log 1.022$$

$$\frac{\log 2.1496}{\log 1.022} = x$$

$$35 = x$$

$$\begin{array}{r} + 1980 \\ \hline 2015 \end{array}$$

29 Matt's rectangular patio measures 9 feet by 12 feet. He wants to increase the patio's dimensions so its area will be twice the area it is now. He plans to increase both the length and the width by the same amount, x. Find x, to the *nearest hundredth of a foot*.

Present area = $9 \times 12 = 108$

Proposed area = $108 \times 2 = 216$

$$(9+x)(12+x) = 216$$

$$108 + 9x + 12x + x^2 = 216$$

$$x^2 + 21x - 108 = 0$$

$a = 1$

$b = 21$

$c = -108$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-21 \pm \sqrt{21^2 - 4(1)(-108)}}{2(1)}$$

$$\frac{-21 \pm \sqrt{873}}{2}$$

$$\frac{-21 + \sqrt{873}}{2}$$

$x = 4.27$

$$\frac{-21 - \sqrt{873}}{2}$$

$x = -25.27$

x must be positive

30 The accompanying table shows the number of new cases reported by the Nassau and Suffolk County Police Crime Stoppers program for the years 2000 through 2002.

Year (x)	New Cases (y)
2000	457
2001	369
2002	353

1
2
3

If $x = 1$ represents the year 2000, and y represents the number of new cases, find the equation of best fit using a power regression, rounding all values to the *nearest thousandth*.

Using this equation, find the estimated number of new cases, to the *nearest whole number*, for the year 2007.

$$y = 451.431 x^{-.243}$$

For the year 2007, $x = 8$

$$y = 451.431(8)^{-.243}$$

$$y = 272$$

31 Dr. Glendon, the school physician in charge of giving sports physicals, has compiled his information and has determined that the probability a student will be on a team is 0.39. Yesterday, Dr. Glendon examined five students chosen at random.
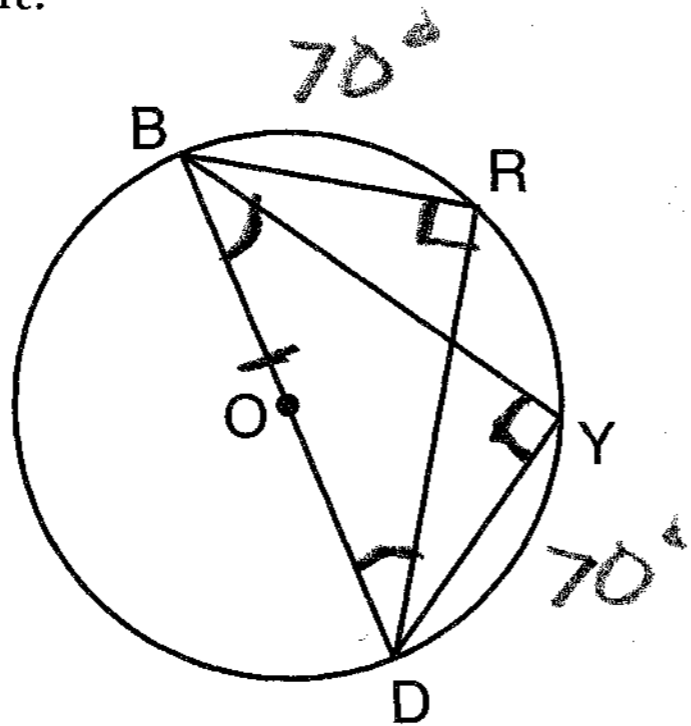
Find, to the *nearest hundredth*, the probability that *at least* four of the five students will be on a team.

Find, to the *nearest hundredth*, the probability that *exactly* one of the five students will *not* be on a team.

$$n = 5$$

$$r = 4, 5$$

$$p = .39$$

$$q = .61$$

$$_5C_4 \, (.39)^4 \, (.61)^1 = .0706$$

$$_5C_5 \, (.39)^5 \, (.61)^0 = \underline{.0090}$$

$$.0796$$

$$.08$$

$$_5C_1 \, (.61)^1 \, (.39)^4 = .07$$

$$n = 5$$

$$r = 1$$

$$p = .61$$

$$q = .39$$

32 In the accompanying diagram, $m\overarc{BR}=70$, $m\overarc{YD}=70$, and $\overline{BOD}$ is the diameter of circle O. Write an explanation or a proof that shows $\triangle RBD$ and $\triangle YDB$ are congruent.

STATEMENT	REASON
1) $\overline{BOD}$ is diameter of circle O	1) Given
2) $\overline{BOD} \cong \overline{DOB}$	2) Reflexive Property
3) $m\overarc{BD} = 180°$	3) A diameter divides a circle into equal halves.
4) $m\angle BRD = 90°$ $m\angle BYD = 90°$	4) The measure of an inscribed $\angle$ is half the measure of its intercepted arc
5) $\angle BRD = m\angle BYD$	5) All right $\angle$s are congruent
6) $m\overarc{BR} = m\overarc{YD} = 70$	6) Given
7) $m\angle BDR = 35$ $m\angle DBY = 35$	7) Same as (4)
8) $m\angle BDR = m\angle DBY$	8) Reflexive Property
9) $\triangle RBD \cong \triangle YDB$	9) AAS

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Perform the indicated operations and simplify completely:

$$\frac{x^2 - 9}{x^2 - 5x} \cdot \frac{5x - x^2}{x^2 - x - 12} \div \frac{x - 4}{x^2 - 8x + 16}$$

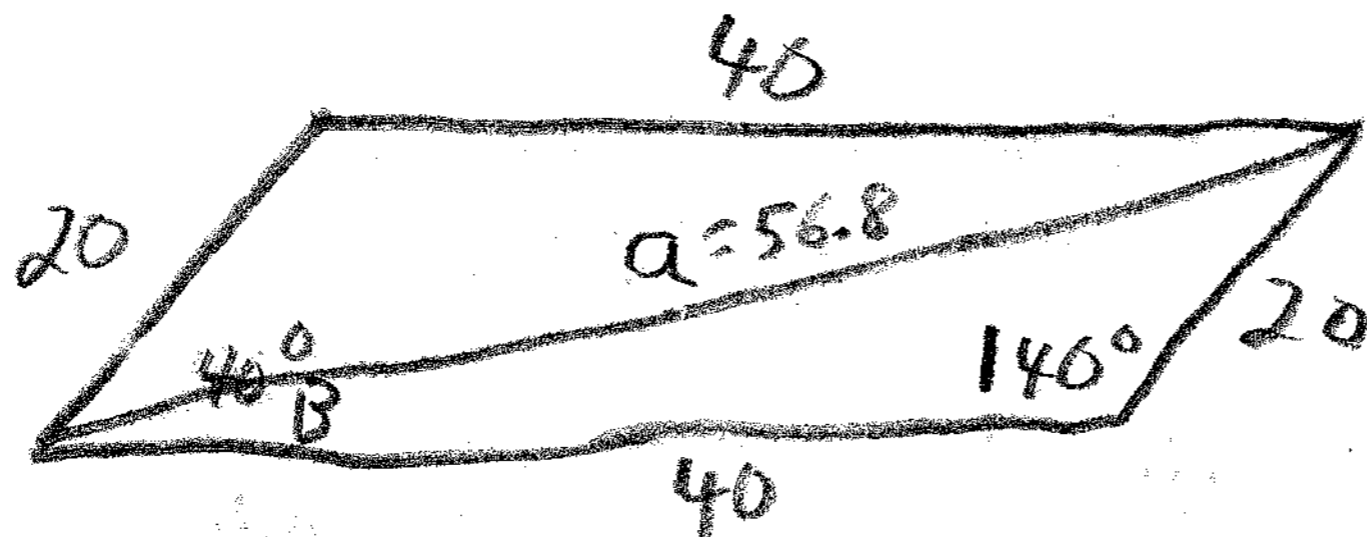$$\frac{(x-3)(x+3)}{x(x-5)} \cdot \frac{x(5-x)}{(x-4)(x+3)} \cdot \frac{(x-4)(x-4)}{x-4}$$

$$\frac{-1(x-3)(5-x)}{(x-5)}$$

$$-(x-3)$$

34 Two forces of 40 pounds and 20 pounds, respectively, act simultaneously on an object. The angle between the two forces is 40°.

Find the magnitude of the resultant, to the *nearest tenth of a pound.*

Find the measure of the angle, to the *nearest degree*, between the resultant and the larger force.

$$180 - 40 = 140$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = 20^2 + 40^2 - 2(20)(40) \cos 140$$
$$a^2 = 3225.67$$
$$a = 56.8$$

$$\frac{56.8}{\sin 140} = \frac{20}{\sin B}$$
$$\sin B = \frac{20 \sin 140}{56.8}$$
$$\sin B = .2263$$
$$B = \sin^{-1} .2263$$
$$B = 13°$$