The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION
INTERMEDIATE ALGEBRA
Monday, August 19, 1963 — 12:30 to 3:30 p.m., only

Name of pupil..........................................................Name of school..........................................................

Name of teacher........................................................................................................................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Factor completely: \(6x + x^2 - x^3\) .......................... 1

2 Express \(\frac{3}{4 - \sqrt{2}}\) as a fraction with a rational denominator. .......................... 2

3 Find the integral value of \(x\) which satisfies the equation \(x = 10 - 3x^2\). .................. 3

4 Find the sum of the integers from 1 through 99, inclusive. .......................... 4

5 If \(y\) varies inversely as the square of \(x\) and \(y = 36\) when \(x = 2\), find the value of \(y\) when \(x = 3\). .......................... 5

6 If the number 0.089 is written in the form \(8.9 \times 10^n\), what is the value of \(n\)? .......................... 6

7 Solve for \(x\): \(\sqrt{x^2 + 3x} = x + 3\) .......................... 7

8 Find the abscissa of the point where the line defined by \(y = 3x - 2\) intersects the \(x\)-axis. .......................... 8

9 Find the value of \((10x)^6 + x^{-4}\) when \(x = 64\). .......................... 9

10 Carry out the indicated operations and write the answer as one term: \(\sqrt{2(3 + \sqrt{5})} - \sqrt{18}\) .......................... 10

11 Write a linear equation which expresses the relation between \(x\) and \(y\) shown in the following table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-2</td>
<td>1</td>
<td>7</td>
<td>16</td>
</tr>
</tbody>
</table>

11

12 What is the greatest value of \(k\) for which the equation \(x^2 + 2x + k = 0\) will have a real root? .......................... 12

[1] [over]
13 Write an equation for the axis of symmetry of the graph of
\[ y = 3x^2 - 12x + 5. \]

14 Write in simplest form the second term only in the expansion of
\[ (3x - 1)^4. \]

15 Find the value of \( \log 985.4 \).

16 Find the value of \( N \) if \( \log N = 9.6286 - 10 \).

17 Find the sum of the infinite geometric progression 10, 9, 8.1, ...

18 A man made a trip of \( m \) miles at an average rate of \( r \) miles per hour. He returned over the same route at an average rate of \( s \) miles per hour. Express in terms of \( m \), \( r \) and \( s \) the total number of hours he spent traveling on the round trip.

19 The sides of a right triangle are 8, 15, 17. What is the value of the tangent of the larger acute angle?

20 Write an equation of the line which passes through the origin and whose slope is equal to the slope of the line \( x + y = 4 \).

21 Solve the formula \( S = \pi a(b + c) \) for \( c \).

22 The units digit of a two-digit number is \( u \). Express the number in terms of \( u \) if the tens digit is twice the units digit.

23 Solve the following pair of equations for \( x \) in terms of \( a \) and \( b \):
\[
\begin{align*}
2x - y &= a \\
3x + 2y &= b
\end{align*}
\]

Directions (24–28) : Write on the line at the right of each of the following the number preceding the expression that best completes the statement or answers the question.

24 \( \log 2x^2 \) is equal to
\[
\begin{align*}
(1) & \ 4 \log x \\
(2) & \ 2(\log 4 + \log x) \\
(3) & \ 2(\log 2 + \log x) \\
(4) & \ \log 2 + 2 \log x
\end{align*}
\]

25 Which of the following is an imaginary number?
\[
\begin{align*}
(1) & \ \sqrt[3]{-8} \\
(2) & \ \sqrt[3]{8} \\
(3) & \ -\sqrt[3]{16} \\
(4) & \ \sqrt{-16}
\end{align*}
\]

26 The graph of the equation \( y^2 - 4 = 3x \) is
\[
\begin{align*}
(1) & \ \text{an ellipse} \\
(2) & \ \text{a circle} \\
(3) & \ \text{a parabola} \\
(4) & \ \text{a hyperbola}
\end{align*}
\]
27 The sum of the roots of the equation \( x^2 + 7x - 5 = 0 \) is
   \[ (1) \, 7 \quad (3) \, 5 \quad (2) \, -7 \quad (4) \, -5 \]

28 The expression \( \frac{x}{x^2 - 9} + \frac{1}{3 - x} \) is equivalent to
   \[ (1) \, \frac{2x + 3}{x^2 - 9} \quad (3) \, \frac{-3}{x^2 - 9} \\
   (2) \, \frac{3}{x^2 - 9} \quad (4) \, \frac{-1}{x^2 - 3} \]

Directions (29–30): For values of \( x \) for which the following statements are defined, indicate whether each of the statements is true for
   \[ (1) \, \text{all real values of } x, \]
   \[ (2) \, \text{some but not all real values of } x, \]
   \[ (3) \, \text{no real values of } x, \]
   by writing on the line at the right the number 1, 2 or 3.

29 \( \sqrt[3]{x^3} = (\sqrt[3]{x})^2 \)  

30 \( \frac{1}{1 + \frac{1}{x}} = 1 \)
Intermediate Algebra — concluded

Part II

Answer four questions from this part. Show all work unless otherwise directed. Only an algebraic solution will be accepted in 32.

31 Answer both a and b:
   a Express in radical form the roots of the equation $2x^2 + x = 2$. [4]
   b Solve algebraically the following set of equations:
      \[
      \begin{align*}
      xy &= -12 \\
      x + y &= 1
      \end{align*}
      \]

32 A man invested two sums of money, the first sum at 6% and the second sum at 4%. The yearly interest from these investments was $280. If he had invested both sums at 5%, his interest would have been $270. Find how much he invested at 6% and how much at 4%. [5, 5]

33 Using logarithms, compute to the nearest integer the value of $\frac{64.2 \sqrt{0.894}}{\tan 27^\circ}$. [10]

34 Write the equation that would be used to solve each of the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]
   a The perimeter of a certain rectangle is 24 inches. If the length is doubled and the width is tripled, then the area is increased by 160 square inches. Find the dimensions of the original rectangle. [3]
   b How many quarts of a 20% salt solution should be added to 20 quarts of a 5% salt solution in order to produce a 10% salt solution? [5]

35 a Plot the graph defined by the equation $xy + 12 = 0$. [6]
   b On the same set of axes used in answer to part a, draw the graph defined by the equation $x + y = 1$. [2]
   c From the graphs made in answer to parts a and b, determine the solutions of the following set of equations:
      \[
      \begin{align*}
      xy + 12 &= 0 \\
      x + y &= 1
      \end{align*}
      \]

*36 Answer either a or b but not both:
   a Solve the following set of equations for $x$, $y$ and $z$ and check:
      \[
      \begin{align*}
      2x + y - z &= 3 \\
      x + 2y + z &= 0 \\
      x - 3y - 2z &= -3
      \end{align*}
      \]
      OR
      \[
      \begin{align*}
      x^3 + 3x^2 = 4x - 12 &= 0
      \end{align*}
      \]
   b Find the roots of the equation $x^3 + 3x^2 - 4x - 12 = 0$. [10]

*This question is based on optional topics in the syllabus.
FOR TEACHERS ONLY

SCORING KEY
INTERMEDIATE ALGEBRA

Monday, August 19, 1963 — 12:30 to 3:30 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil’s work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit unless an equation is written in 11, 13 and 20. For questions 24-30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$x(2 + x)(3 - x)$</td>
<td>(13)</td>
</tr>
<tr>
<td>(2)</td>
<td>$\frac{3(4 + \sqrt{2})}{14}$</td>
<td>(14)</td>
</tr>
<tr>
<td>(3)</td>
<td>$-2$</td>
<td>(15)</td>
</tr>
<tr>
<td>(4)</td>
<td>4,950</td>
<td>(16)</td>
</tr>
<tr>
<td>(5)</td>
<td>16</td>
<td>(17)</td>
</tr>
<tr>
<td>(6)</td>
<td>$-2$</td>
<td>(18)</td>
</tr>
<tr>
<td>(7)</td>
<td>$-3$</td>
<td>(19)</td>
</tr>
<tr>
<td>(8)</td>
<td>$\frac{5}{4}$</td>
<td>(20)</td>
</tr>
<tr>
<td>(9)</td>
<td>1</td>
<td>(21)</td>
</tr>
<tr>
<td>(10)</td>
<td>21a</td>
<td>(22)</td>
</tr>
<tr>
<td>(11)</td>
<td>$y = 3x - 2$</td>
<td>(23)</td>
</tr>
<tr>
<td>(12)</td>
<td>1</td>
<td>(24)</td>
</tr>
<tr>
<td>(25)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>(26)</td>
<td>3</td>
<td>[OVER]</td>
</tr>
</tbody>
</table>
Please refer to the Department’s pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) \( a \frac{-1 \pm \sqrt{17}}{4} \) \[4\]

\( b \ (-3,4), (4,-3) \) \[6\]

(32) $3,200 at 6\% ; $2,200 at 4\%$

(33) 119

(34) \( a \) Let \( x \) = number of inches in original length

\( 6x(12 - x) = x(12 - x) + 160 \) \[5\]

\( b \) Let \( x \) = number of quarts added

\( .2x + 1 = .1(x + 20) \) \[5\]

(35) \( c \ (4,-3), (-3,4) \) \[2\]

(36) \( a \ (-2,3,-4) \) \[8\]

Check \[2\]

\( b \ 2,-2,-3 \) \[10\]