The University of the State of New York

322nd High School Examination

INTERMEDIATE ALGEBRA

Tuesday, August 24, 1954 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in intermediate algebra previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1954 or number and length in minutes of lessons taken in the summer of 1954 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1954 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from this part. All work, including computation, should be shown.

26 Find the nearest tenth the roots of the equation $2x - \frac{5}{x} - 6 = 0$. [10]

27 Solve the following system of equations and check: $[8, 2]$

\[
\begin{align*}
2x^2 - xy &= 2 \\
x - y + 1 &= 0
\end{align*}
\]

28 Given the formula $K = \frac{a^2}{b\sqrt{\sin C}}$. Using logarithms, find $K$ to the nearest integer when $a = 6.84$, $b = 3.07$ and $C = 54^\circ$. [10]

29 $a$ Draw the graph of the equation $y = -x^2 + 4x + 1$ from $x = -2$ to $x = 6$ inclusive. [6]

$b$ Write the equation of the axis of symmetry of the graph drawn in answer to $a$. [1]

$c$ Write the coordinates of the maximum point. [1]

$d$ From the graph drawn in answer to $a$, find to the nearest tenth the roots of the equation $-x^2 + 4x + 1 = 0$. [2]

*30 Answer either $a$ or $b$: [10]

$a$ Solve the following system of equations for $x$, $y$ and $z$:

\[
\begin{align*}
3x - y + z &= 1 \\
2x &= z = -4 \\
x + 2y + 2z &= -1
\end{align*}
\]

$b$ Find the roots of the equation $3x^3 - 2x^2 - 12x + 8 = 0$.

* This question is based on optional topics in the syllabus and may be used in place of any question in either part II or part III.

[1] [over]
31 When each one of the numbers listed in column I is substituted for $k$ in the equation $x^2 - 4x + k = 0$, the roots of the resulting equation are correctly described by only one of the statements in column II. List the numbers 1-5 on your answer paper and after each number write one of the letters $a$-$h$ to indicate the statement that correctly describes the roots of the corresponding equation. [10]

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $k = -5$</td>
<td>$a$ The roots are real, equal and rational.</td>
</tr>
<tr>
<td>(2) $k = -4$</td>
<td>$b$ The roots are real, unequal and irrational.</td>
</tr>
<tr>
<td>(3) $k = 0$</td>
<td>$c$ The roots are imaginary.</td>
</tr>
<tr>
<td>(4) $k = 4$</td>
<td>$d$ One root is zero and the other root is positive.</td>
</tr>
<tr>
<td>(5) $k = 5$</td>
<td>$e$ Both roots are zero.</td>
</tr>
<tr>
<td></td>
<td>$f$ Both roots are negative and equal.</td>
</tr>
<tr>
<td></td>
<td>$g$ Both roots are positive and unequal.</td>
</tr>
<tr>
<td></td>
<td>$h$ One root is positive, one is negative and both are rational.</td>
</tr>
</tbody>
</table>

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of equations is not required.]

- $a$ A merchant sold an article for $38.25$ at a loss of 15% of the cost. What was the cost of the article to the merchant? [4]
- $b$ An airplane is 1500 miles from its destination. If it continues at its present rate, it will arrive at its destination one hour late. If its rate is increased by 50 miles per hour, it will arrive on time. Find its present rate. [6]

33 How many pounds of pure salt should be added to 90 pounds of a 6% solution of salt and water to change it to a solution that is 10% salt? [All work, including computation, should be shown.] [10]

34 The length of a rectangle is $l$ and the width is $w$. If the length of this rectangle is decreased by a certain amount, $x$, and its width is increased by the same amount, then the area of the resulting rectangle is equal to the area of the original rectangle.

- $a$ Express the area of the resulting rectangle in terms of $l$, $w$ and $x$. [2]
- $b$ Write an equation that may be used to find $x$. [2]
- $c$ Solve this equation for $x$ in terms of $l$ and $w$. [All work, including computation, should be shown.] [6]
Intermediate Algebra — continued

Fill in the following lines:

Name of pupil...........................................Name of school...........................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Factor $3x^2 + x - 10$.

2 Express as a single term the sum of $\frac{1}{2} \sqrt{-9}$ and 6i.

3 Simplify: $\frac{\frac{c}{3} - 1}{\frac{c}{3} + \frac{1}{c}}$

4 Reduce to lowest terms: $\frac{x^2 - y^2}{y - x}$

5 Find the value of $(2x)^0 + 2x^4$.

6 Find the value of $x^{-\frac{3}{4}}$ when $x = 27$.

7 Express as a single power: $2^a \cdot 4^x$

8 Express $\frac{7}{2\sqrt{5}}$ as an equivalent fraction with a rational denominator.

9 Write a linear equation expressing the relationship between $x$ and $y$ shown in the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$0$</th>
<th>$6$</th>
<th>$9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-3$</td>
<td>$0$</td>
<td>$3$</td>
<td>$5$</td>
</tr>
</tbody>
</table>

10 During a certain year, a man has an income of $I$ dollars. He is allowed to deduct $x$ dollars from his income, the remainder being subject to a tax of 20%. Write an expression for the amount of his tax in terms of $I$ and $x$.

11 Solve the formula $a = \frac{b - c}{c}$ for $c$.

12 If $x$ varies directly as $y$ and $y = 3$ when $x = 32$, find $y$ when $x = 80$.

13 Solve for $x$: $2\sqrt{x - 1} + 3 = 7$ [3]

[over]
14 Find the logarithm of 0.06432

15 If \( \log y = 0.5531 \), find \( y \) to the nearest thousandth.

16 If \( \log x^4 = 1.7320 \), find \( \log x \).

17 Write the first two terms in the expansion of \((x - 3y)^5\).

18 Write the equation of the line whose slope is 7 and which passes through the point \((0, 1)\).

19 Find the sum of the roots of the equation \(3x^2 - 7x + 5 = 0\).

20 At a point 105 feet from the foot of a vertical pole on level ground, the angle of elevation of the top of the pole is 22°. Find the height of the pole to the nearest foot.

21 If the 6th term of an arithmetic progression is 23 and the ninth term is 35, find the common difference.

22 Insert two geometric means between 2 and 128.

23 Find the sum of the infinite series \(3 + \frac{3}{2} + \frac{3}{4} + \ldots\)

Directions (24 and 25): Indicate the correct answer for each question by writing the letter \(a\), \(b\) or \(c\) on the line at the right.

24 Which is correct if \( \log K = \log x - \log y \)? (a) \(K = x - y\) (b) \(K = xy\) (c) \(K = \frac{x}{y}\)  

25 Which is an equation of an ellipse? (a) \(y^2 = 16 + 4x^2\) (b) \(y = 4x^2 - 16\) (c) \(y^2 = 16 - 4x^2\)
Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil’s work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 24 and 25, allow credit if the pupil has written the correct expression instead of the letter c.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$(3x - 5)(x + 2)$</td>
<td>(12)</td>
<td>$\frac{7}{2}$</td>
</tr>
<tr>
<td>(2)</td>
<td>$\frac{c^2 - 3}{c^2 + 3}$</td>
<td>(13)</td>
<td>5</td>
</tr>
<tr>
<td>(3)</td>
<td>$(y + x)$</td>
<td>(14)</td>
<td>$8.8083 - 10$</td>
</tr>
<tr>
<td>(4)</td>
<td>$3$</td>
<td>(15)</td>
<td>3.573</td>
</tr>
<tr>
<td>(5)</td>
<td>$\frac{1}{9}$</td>
<td>(16)</td>
<td>.4330</td>
</tr>
<tr>
<td>(6)</td>
<td>$2^x$ or $8^x$</td>
<td>(17)</td>
<td>$x^4 - 15x^2y$</td>
</tr>
<tr>
<td>(7)</td>
<td>$\frac{7\sqrt{5}}{10}$</td>
<td>(18)</td>
<td>$y = 7x + 1$</td>
</tr>
<tr>
<td>(8)</td>
<td>$3y = 2x - 3$</td>
<td>(19)</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>(9)</td>
<td>$\frac{I - x}{5}$</td>
<td>(20)</td>
<td>42</td>
</tr>
<tr>
<td>(10)</td>
<td>$c = \frac{b}{a + 1}$</td>
<td>(21)</td>
<td>4</td>
</tr>
<tr>
<td>(11)</td>
<td></td>
<td>(22)</td>
<td>8, 32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(23)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(24)</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25)</td>
<td>c</td>
</tr>
</tbody>
</table>