The University of the State of New York
319th High School Examination
INTERMEDIATE ALGEBRA
Wednesday, August 19, 1953 — 12 m. to 3 p. m., only

Instructions
Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in intermediate algebra previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1953 or number and length in minutes of lessons taken in the summer of 1953 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1953 or an equivalent program of tutoring approved in advance by the Department is required.

Part II
Answer three questions from part II.

26 Solve the following system of equations and check: [8, 2]
   \[ x^2 + y^2 = 25 \]
   \[ 2x - y + 5 = 0 \]

27 Given the formula \( h = \sqrt[3]{\frac{k \cos^2 A}{I}} \). Using logarithms, find \( h \) to the nearest hundredth, when \( k = 518 \), \( A = 39^\circ \) and \( I = 862 \). [10]

28 One leg of a right triangle is 3 inches longer than the other leg and the hypotenuse is 7 inches. Find to the nearest tenth of an inch the length of the shorter leg. [2, 8]

29 a Draw the graph of the equation \( y = x(8 - 2x) \) from \( x = -1 \) to \( x = 5 \). [6]
   b Write the equation of the axis of symmetry of the graph drawn in answer to a. [2]
   c Find the maximum value of \( y \) in the graph drawn in answer to a. [2]

*30 Solve the following system of equations for \( x, y, \) and \( z \): [10]
   \[ 3x + 2y - 6z = -4 \]
   \[ -x + y = -5 \]
   \[ 4x + 2y + 3z = 4 \]

*31 Find the roots of the equation \( 2x^3 - 5x^2 - 9x + 18 = 0 \). [10]

* This question is based upon one of the optional topics in the syllabus.
Part III
Answer two questions from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a Carl drove a distance of 120 miles in \( 3\frac{1}{2} \) hours. He was held back by heavy traffic for the first half of the distance but averaged 10 miles per hour faster on the second half. What was his average rate over the first half of the road? [5]

b Jay, working alone, takes twice as long as Howard to paint a house. Howard starts alone and, after half the job is done, he is joined by Jay. They work together for 6 days to finish the painting. How long would it have taken each to paint the house, working alone? [5]

33 A picture measuring 12 inches by 15 inches is surrounded by a frame of uniform width. If the area of the picture is \( \frac{2}{3} \) of the total area of the picture and frame together, how wide is the frame? [6, 4]

34 Three numbers form an arithmetic progression whose common difference is 3. If the first number remains unchanged, the second is increased by 3 and the third is increased by 9, the resulting numbers, taken in that order, form a geometric progression. Find the first term. [6, 4]

35 Each of the curves described in column I is the graph of one of the equations listed in column II. Write the numbers (1) through (5) on your answer paper and after each number write the letter that indicates the corresponding equation. [10]

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) A circle whose center is the origin</td>
<td>( a \quad x^2 + 4y^2 = 12 )</td>
</tr>
<tr>
<td>(2) A parabola whose axis of symmetry is the y-axis</td>
<td>( b \quad y^2 - 2x^2 = 5 )</td>
</tr>
<tr>
<td>(3) An ellipse whose major axis is on the x-axis</td>
<td>( c \quad y = x^2 + 7 )</td>
</tr>
<tr>
<td>(4) An hyperbola that crosses the y-axis</td>
<td>( d \quad x^2 + y^2 = 30 )</td>
</tr>
<tr>
<td>(5) A parabola that is tangent to the x-axis</td>
<td>( e \quad x^2 = y^2 = 25 )</td>
</tr>
<tr>
<td></td>
<td>( f \quad 2x^2 + y^2 = 6 )</td>
</tr>
<tr>
<td></td>
<td>( g \quad y = x^2 + 4x + 4 )</td>
</tr>
<tr>
<td></td>
<td>( h \quad y = x^2 - 8x )</td>
</tr>
</tbody>
</table>

[2]
INTERMEDIATE ALGEBRA

Fill in the following lines:

Name of pupil .............................................Name of school................................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Find the positive root of the equation \( x^2 - x - 12 = 0 \).

2. Express \( 2\sqrt{-3} \) in terms of \( i \).

3. Solve the equation \( \sqrt{x - 5} - 3 = 0 \) for \( x \).

4. The discriminant of a quadratic equation with integral coefficients is 27. Are the roots rational or are they irrational?

5. Find the average of the numbers \( a, b, c \).

6. Find the logarithm of .04566

7. If \( \log N = 1.7569 \), find \( N \) to the nearest hundredth.

8. One leg of a right triangle is 9 and the hypotenuse is 13. Find to the nearest degree the angle opposite the given leg.

9. Express \( \frac{2}{4 - \sqrt{3}} \) as an equivalent fraction with a rational denominator.

10. Solve for \( n \) the equation \( an = bn + c \).

11. Write the equation of the line whose slope is \( \frac{3}{5} \) and whose \( y \)-intercept is 2.

12. Write in simplest form: \( 2^a \times 2^n \)

13. Find the positive geometric mean between \(-8\) and \(-2\).

14. Find the sum of the infinite series \( 2 + \frac{3}{5} + \frac{3}{5} + \ldots \)  

[3]
15 Write in simplest form the first three terms of the expansion of 
\((x - y)^8\).

16 Find the value of \(3a^0 + 4a^{-1}\) when \(a = 4\).

17 Find the sum of the roots of the equation \(2x^2 - 3x + 6 = 0\).

18 If \(xy = 12\) and \(x\) is positive, does \(y\) increase or does \(y\) decrease as \(x\) increases?

19 Write in simplest form the sum of \(\frac{x - 2}{-2}\) and \(\frac{x - 3}{3}\).

20 If \(h\) varies directly as \(s\) and \(h = 2.4\) when \(s = 3\), find \(h\) when \(s = 8\).

Directions (21–22): Indicate the correct completion for each of the following by writing on the line at the right the letter \(a\), \(b\) or \(c\).

21 A solution of salt and water that is \(p\%\) salt weighs \(x\) pounds. The number of pounds of salt in the solution is
\( (a) px\) \( (b) 0.01px\) \( (c) 0.01px\).

22 \(\log 2x^3\) is equal to
\( (a) 6 \log x\) \( (b) 3 \log 2x\) \( (c) \log 2 + 3 \log x\).

23 If \(320,000,000,000 = 3.2 \times 10^n\), what is the value of \(n\)?

24 \(A\) has \(x\) dollars and \(B\) has \(y\) dollars. Write an equation that indicates that if \(A\) gives \(z\) dollars to \(B\), then \(A\) and \(B\) will have the same number of dollars.

25 If the statement \((\sqrt{x})^2 = x\) is true for all values of \(x\), find the value of \((\sqrt{-1})^2\).