The University of the State of New York
307th High School Examination

INTERMEDIATE ALGEBRA

Tuesday, August 23, 1949 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) names of schools where you have studied, (b) number of weeks and recitations a week in intermediate algebra previously to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1949 or number and length in minutes of lessons taken in the summer of 1949 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester’s work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1949 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Find, to the nearest tenth, the roots of the equation: \(2x^2 - 5x - 4 = 0\) \([10]\)

27 a. Draw the graph of \(y = x^3 - 4x - 4\) from \(x = -1\) to + 5 inclusive. \([7]\)
   b. From the graph made in answer to a:
      (1) Estimate to the nearest tenth the roots of \(x^3 - 4x - 4 = 0\) \([2]\)
      (2) For what value of \(y\) are the roots of \(y = x^3 - 4x - 4\) equal? \([1]\)

28 Using logarithms, find, to the nearest tenth, the value of \(\frac{25.6 \cos 36^\circ}{\sqrt[3]{.827}}\) \([10]\)

29 Solve the following system of equations, correctly group your answers and check one set:
\[
x^2 + y^2 - y = 13
x + y = 3
\]
\([7, 2, 1]\)

*30 Solve for \(x\), \(y\) and \(z\):
\[
x + 2y + 3z = -5
2x + 3y - 4z = -8
3x - 4y + 5z = 5
\]
\([10]\)

* This question is based on one of the optional topics in the syllabus.

[1] \[\text{over}\]
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Part III
Answer one question from part III.

31. Write the equation that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]
   a. A can do a piece of work in 4 hours and B can do it in 5 hours. A works alone for 1 hour. How long will it take B, working alone, to finish the job? [5]
   b. A dairyman wishes to mix milk containing 5% butterfat and cream containing 75% butterfat in such a way that a total mixture of 56 pounds shall contain 50% butterfat. How much of each kind should he use? [5]

32. A man sold some shares of stock for a total of $400. When the price dropped $5 per share he reinvested the $400 to purchase 4 more shares than he had sold. How many shares did he sell? [5, 5]

Part IV
Answer one question from part IV.

33. Given the equation \( x^2 + x - K = 0 \) whose discriminant is represented by \( D \).
   a. Write an equation expressing the relationship between \( D \) and \( K \). [3]
   b. Draw the graph of the equation given in answer to a. [Use \( D \) as the vertical axis.] [3]
   c. Explain how it is possible to determine from the graph the values of \( K \) for which the roots of \( x^2 + x - K = 0 \) are
      (1) real and unequal [2]
      (2) imaginary [2]

34. Using the formula \( A = P (1 + r)^n \), find, to the nearest per cent, the rate at which $400 must be invested to amount to $496 in 12 years, interest compounded annually. [Use logarithms.] [10]
Fill in the following lines:

Name of pupil ............................................. Name of school ..........................................

**Part I**

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Reduce to lowest terms $\frac{5x - 5y}{y - x}$

2. Express the sum of $\sqrt{-9}$ and $\sqrt{-25}$ in terms of $i$. [Answer must be left in simplest form.]

3. Factor $x^2 - 1$

4. Solve the equation $\sqrt{2x + 5} - 3 = 0$

5. Find the logarithm of $678.4$

6. Find the number whose logarithm is $1.8428$

7. If $\log x = a$, express $\log x^3$ in terms of $a$.

8. Find the value of $4^2 \times 4^{-1} \times 4^{\frac{1}{2}}$

9. Write the linear equation expressing the relation between $x$ and $y$ shown in the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>5</td>
<td>11</td>
<td>20</td>
</tr>
</tbody>
</table>

10. Write in the form $x^2 + px + q = 0$ the equation whose roots are 1 and $-3$.

11. What is the name of the graph of $x^2 - y^2 = 4$?

12. Write the equation of the line which passes through the origin and is parallel to the line whose equation is $y = 3x - 1$.

13. If $x$ varies directly as $y$ and $x$ is 12 when $y$ is 2, find the value of $x$ when $y$ is 3.

14. Find the 15th term of the progression $-1, -1\frac{1}{2}, -2, \ldots$.

15. Find the positive geometric mean between 8 and 50.

16. Find the sum of the infinite progression $0.2, 0.02, 0.002, \ldots$.

17. Find, to the nearest degree, the angle of elevation of the sun when a 25-foot vertical pole casts a shadow 6.5 feet long.

18. Express in simplest form $\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{a}{b}}$

19. If a train goes $m$ miles in $x$ minutes, how far will it go in one hour?

20. Solve for $x$ the equation $3^x = 9^2$

21. Write in simplest form the second term of the expansion of $(x + \frac{1}{x})^8$

22. Write the fraction $\frac{1}{3 - \sqrt{2}}$ with a rational denominator.

[3] [OVER]
Directions (questions 23-25) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

23 The sum of the roots of the equation $2x^2 - 4x + 7 = 0$ is
(a) $+4$  
(b) $+2$  
(c) $-2$  

24 If the discriminant of a quadratic equation is $-9$, then the roots of the equation are
(a) real and rational  
(b) real and irrational  
(c) imaginary  

25 When drawn on the same set of axes, the graphs of $y = 2x - 2$ and $xy = 24$
(a) intersect in two points  
(b) intersect in one point  
(c) do not intersect