The University of the State of New York

301st HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, August 20, 1947 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) names of schools where you have studied, (b) number of weeks and recitations a week in intermediate algebra previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1947 or number and length in minutes of lessons taken in the summer of 1947 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1947 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Find, correct to the nearest tenth, the roots of the equation $2x^2 - 7x + 2 = 0$ \hspace{1cm} [10]

27 Solve the following set of equations, group your answers and check one set: \hspace{1cm} [7, 2, 1]

\[
\begin{align*}
x^2 + y^2 &= 23 \\
2x^2 - y^2 &= 25
\end{align*}
\]

28 a On the same set of axes draw the graphs of $xy = 12$ and $2x - y = 2$ \hspace{1cm} [6, 2]

b From the graphs made in answer to a, determine the values of $x$ and $y$ common to the equations $xy = 12$ and $2x - y = 2$ \hspace{1cm} [2]

29 By logarithms, find, correct to the nearest thousandth, the value of $\frac{4.19 \times \sqrt[3]{0.473}}{24.4}$ \hspace{1cm} [10]

30 Solve for $x, y$ and $z$: \hspace{1cm} [10]

\[
\begin{align*}
2x + 3y + 4z &= 5 \\
x - 2y - z &= 3 \\
x + y + 3z &= 4
\end{align*}
\]

31 Find the factors of $x^4 - 7x^2 - 6x$ \hspace{1cm} [10]

* This question is based on one of the optional topics in the syllabus.

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Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a A 4% solution of salt and water weighs 200 pounds. How many pounds of water must be evaporated to make it a 6% solution? [5]

b A photographer left his car at the foot of a mountain and walked to the top at an average rate of 2¾ miles per hour. After taking pictures for 1 hour, he returned over the same trail at an average rate of 3½ miles per hour. He arrived back at his car 6 hours after he left it. How long is the trail to the top of the mountain? [5]

33 A man bought a number of gallons of maple syrup for $80. He kept 4 gallons for his family to use and sold the rest for $80. As a result, the selling price per gallon was $1 more than the purchase price per gallon. How many gallons did he buy? [6, 4]

Part IV

Answer one question from part IV.

34 Given the equation: \(2x^2 + kx + 2 = 0\)

a In questions (1), (2) and (3), the value of \(k\) is increasing from 0 to + 6. In each case select the correct answer, (a), (b) or (c).

(1) The sum of the roots (a) increases (b) decreases (c) remains the same [2]

(2) The product of the roots (a) increases (b) decreases (c) remains the same [1]

(3) The value of the discriminant (a) increases (b) decreases (c) remains the same [1]

b If the graph of \(y = 2x^2 + kx + 2\) were drawn, give a value of \(k\) between 0 and 6 inclusive for which the graph of \(y = 2x^2 + kx + 2\) would

(1) Intersect the \(x\) axis [2]

(2) Be tangent to the \(x\) axis [2]

(3) Have no point in common with the \(x\) axis [2]

35 Henry and Frank can do a piece of work in \(b\) hours and \(m\) hours respectively if working alone. After they had worked together for \(r\) hours, Henry was called away and Frank completed the work.

a What fractional part of the work was done in \(r\) hours by (1) Henry, (2) Frank? [2]

b If \(x\) represents the number of additional hours required by Frank to complete the work, what fractional part did he do in \(x\) hours? [1]

c Write the equation that can be used to find \(x\) in terms of \(b, m\) and \(r\). [4]

d Solve the equation for \(x\). [3]
Fill in the following lines:

Name of school..............................................Name of pupil..............................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the value of \(2x^0y^4\) if \(x = 2\) and \(y = 16\)
2 Express the sum of \(\sqrt{-16}\) and \(\sqrt{-25}\) in terms of \(i\).
3 Is the following statement true for all values of \(x\) and \(y\)?
   \(\sqrt{9x^2 + y^2} = 3x + y\)
4 Solve for \(x\) the equation \(\sqrt{x - 2} - 3 = 0\)
5 Find the logarithm of 2.413
6 Find, correct to the nearest tenth, the number whose logarithm is 2.4317
7 If \(\log n = a\) and \(\log t = b\), express \(\log \frac{n}{t}\) in terms of \(a\) and \(b\).
8 One side of a rectangle is 12 and it makes an angle of 40° with the diagonal. Find, correct to the nearest tenth, the other side of the rectangle.
9 If \(A\) varies directly as \(h\) and \(A\) equals 24 when \(h\) equals 7, find \(A\) when \(h\) equals 21.
10 Does the value of \(1 - \frac{3}{x + 2}\) increase or does it decrease as \(x\) increases from 0?
11 Express as a single fraction in its lowest terms: \(\frac{5}{2x - 2y} + \frac{3}{4x}\)
12 Insert two arithmetic means between 4 and \(-14\).
13 A formula for finding the sum of a geometric progression in terms of \(a\), \(n\) and \(r\) is \(S = \ldots\)
14 Find the sum of the first six terms of the arithmetic progression whose first term is 3 and whose sixth term is 23.
15 A lending library charges 10 cents for the first three days a book is kept out and 2 cents a day for each additional day. A formula for the number of cents \((C)\) charged for a book which was kept out for \(d\) days, \(d\) being greater than 3, is \(C = \ldots\)
16 Find the smaller value of \(x\) which satisfies the equation
   \(2x^2 - 7x + 3 = 0\)
17 Write in the form \(x^2 + px + q = 0\) the equation whose roots are \(-3\) and \(-4\).
18 Find the product of the roots of the equation \(x^2 - 3x = 0\)
19 Write the discriminant of the quadratic equation \(x^2 + px + q = 0\)
20 Write in simplest form the first three terms of \((a - 2)^6\)
21 Write the equation of the straight line which crosses the y axis at the point \((0, 5)\) and is parallel to the line whose equation is \(y = 2x + 7\)

[3]
Directions (questions 22–25) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

22 When solved for $t$, the formula $n = \frac{t + \frac{5b}{t}}{t}$ becomes

(a) $t = \frac{5b}{n - 1}$  
(b) $t = \frac{5b}{n}$  
(c) $t = \frac{5b}{n + 1}$

23 The number .000096 may be written

(a) $9.6 \times 10^{-4}$  
(b) $9.6 \times 10^5$  
(c) $9.6 \times 10^{-5}$

24 The graph of the equation $3x^2 - 4y^2 = 16$ is

(a) an ellipse  
(b) a hyperbola  
(c) a circle

25 The fraction $\frac{6}{\sqrt{2} - \sqrt{3}}$ is equal to

(a) $-6\sqrt{2} + \sqrt{3}$  
(b) $-6\sqrt{2} + 6\sqrt{3}$  
(c) $-6\sqrt{2} - 6\sqrt{3}$