Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV, (a) names of schools where you have studied, (b) number of weeks and recitations a week in intermediate algebra previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1940.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1940 is required.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–8) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b, c or d.

1. If the discriminant of a quadratic equation is \(-4\), the roots of the equation are (a) rational and equal, (b) rational and unequal, (c) irrational and unequal or (d) imaginary.

2. The graph of \(x^2 + 4y^2 = 25\) is (a) an ellipse, (b) a hyperbola, (c) a parabola or (d) a circle.

3. Two geometric means between 2 and 128 are (a) 16 and 64, (b) 44 and 86, (c) 8 and 32 or (d) 8 and 32.

4. \(\log \frac{a^2}{b}\) equals (a) \(2 \log a \div \log b\), (b) \(\frac{1}{2} \log a - \log b\), (c) \(2 \log a - \log b\) or (d) \(2 \log a + \log b\).

5. Using the formula \(A = P (1 + rt)\), express \(r\) in terms of \(A\), \(P\) and \(t\).

6. Solve for \(x\) the equation \(\sqrt{6x + 7} = 3 = 0\)

7. Express \(\frac{5}{3 + \sqrt{2}}\) as an equivalent fraction with a rational denominator.

8. Combine \(2\sqrt{18} - \frac{3}{2}\sqrt{2} + \sqrt{3}\) into a single term.

9. Write the quadratic equation the sum of whose roots is 6 and the product of whose roots is 7.

10. Of what binomial are \(x^n + 1\) and \(x^n - 1\) factors?

11. Complete the formula \(S = \ldots\) for a geometric progression in terms of \(a\), \(r\) and \(n\).

12. Find the 37th term of the progression 3, 4\(\frac{1}{2}\), 6, ....

13. Write the first three terms in the expansion of \((a - b)^2\).

14. Find the two values of \(y\) corresponding to the given values of \(x\) that could be used in plotting the graph of \(x + 3y = 5\)

\[
\begin{array}{c|cc}
  x & -1 & 5 \\
  y &   &   \\
\end{array}
\]

15. Which, if any, of the sets of \(x\) and \(y\) values given below are not roots of \(x^2 - 3x = y\)?

\[
\begin{array}{c|ccc}
  x & -1 & 2 & 3 \\
  y & 4 & 2 & 0 \\
\end{array}
\]

[over]
16. At a point 30 feet from the base of a tree the angle of elevation of its top is 53°. Find, correct to the nearest foot, the height of the tree.

17. Find the logarithm of .05506.

18. Find, correct to the nearest integer, the number whose logarithm is 3.6593.

19. In the equation \( y = 2 - \frac{1}{x} \), does \( y \) increase or decrease as \( x \) increases from 1?

20. Find the value of \( x^4 + 3x^3 \) when \( x = 8 \).

21. Simplify \( (1 - \frac{2}{x^2 + 1}) + (x - 1) \).

22. Write an equation expressing the relation between \( x \) and \( y \) shown in the table below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

23. For what value of \( x \) do \( x + 4 \) and \( 2x - 3 \) represent the same number?

24. What is the \( y \) intercept of the line represented by the equation \( 2y = x + 4 \)?

25. A dealer bought radios for $A apiece. He sold them at a price that gave him a profit of 40% on the selling price. Using \( x \) to represent the selling price, write an equation that expresses the relation between cost, profit and selling price.
Solve the equation $3x^2 - 2x - 6 = 0$ for values of $x$ correct to the nearest tenth. [10]

27 Solve the following simultaneous equations and check one set of answers:
   
   \[
   \begin{align*}
   x^2 - 2y &= 11 \\
   x &= y + 4 \\
   \end{align*}
   \]

   [8, 2]

28 Find, correct to the nearest thousandth, the value of $\frac{408 \times \sqrt{3}}{5\tan 16^\circ}$

[Use logarithms.] [10]

29 In a certain school system the salary scale for teachers starts at $1400 and provides for a yearly increase of $75 for the next five years. Miss A, starting at the minimum salary, plans on making $1350 cover her entire expenses for each year. How much will she be able to save if she stays five years? [Use formula in solution.] [10]

30 a Plot the graph of $x = y^2 - 2y$ from $y = -2$ to $y = 4$ inclusive. [6]
   b On the same set of axes, plot the graph of $2y = 3x$. [2]
   c From the graphs made in answer to $a$ and $b$, find the values of $x$ and $y$ common to the two equations. [2]

Group III

Answer one question from this group.

31 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

   a The units digit of a two-digit number is twice the tens digit. If the digits are reversed, the resulting number exceeds the original number by 27. Find the original number. [5]
   
   b How many pounds of coffee worth 16¢ a pound must be mixed with 20 pounds of coffee worth 25¢ a pound to have a mixture worth 18¢ a pound? [5]

32 The distance from $A$ to $B$ is 90 miles. An autoist starting from $A$ drives at a uniform rate per hour until he is 15 miles from $B$, when he slows down to one third of his initial rate. He reaches $B$ 40 minutes later than he had planned. Find his original rate. [7, 3]

Group IV

Answer one question from this group.

33 a Find the value of $x$ if $\log x = 2 + \log 3$. [3]
   
   b A circular pool is surrounded by a walk of uniform width. The difference in the circumferences of the two circles is 22 feet. Find the width of the walk. [Use $\pi = \frac{\sqrt{e}}{2}$] [7]

34 a Is each of the lettered items, (a), (b), (c) and (d), in the following problem necessary for the solution? Explain your answer. [Solution of the problem is not required.]

   An open tank having (a) a capacity of 15 gallons contains (b) 10 gallons of a solution of salt and water which is (c) 8% salt. How many gallons of water must be evaporated so that the solution shall be (d) 12% salt? [3]
   
   b Three hundred rods of fencing are to be used to inclose the largest possible rectangular yard $ABCD$. The fence on side $AB$ is to be made double height. [7]
   
   (1) If $x$ equals the length in rods of side $AB$, express in terms of $x$ the length of side $AD$.
   
   (2) If $y$ equals the area of $ABCD$, express $y$ in terms of $x$.
   
   (3) Graph the equation written in answer to (2) from $x = 0$ to $x = 100$ inclusive, at intervals of 20.
   
   (4) Estimate from the graph the maximum area that the yard can have.