The University of the State of New York

321st High School Examination

INTERMEDIATE ALGEBRA

Wednesday, June 23, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II. All work, including computation, should be shown.

26 Given the equation $2x^2 - 5x - 8 = 0$.

a Solve the equation for x. [Answers may be left in radical form.] [8]

b Using the relation between the roots and the coefficients of the given equation, check the results found in answer to *a*. [2]

27 Solve the following system of equations and check the answers: [8, 2]

$$x^2 + y^2 - 10y = 24$$
$$y = x - 2$$

28 Using logarithms, find to the *nearest hundredth* the value of: [10]

$$\sqrt[8]{\frac{87.2 \sin 43°}{1970}}$$

29 Solve graphically the following system of equations: [6, 2, 2]

$$xy = 12$$
$$y = x + 4$$

*30 Answer *either a* or *b*:

a Three numbers, x, y and z, are such that their sum is 19. The first diminished by the sum of the second and third is 9, and the sum of the first and second diminished by the third is 25.

(1) Write the three equations that can be used to find the numbers. [3]

(2) Find the numbers and check. [6, 1]

b If P dollars is invested at $r\%$ with interest compounded annually for n years, the amount, A, is given by the formula $A = P(1 + r)^n$.

(1) Show that n is equal to $\dfrac{\log A - \log P}{\log (1 + r)}$ [5]

(2) Find n when $P = \$600$, $A = \$857$, and $r = 2\%$. [5]

* This question is based on optional topics in the syllabus and may be used in place of any question in either part II or part III.

Part III

Answer two questions from part III.

31 Write the equations that would be used to solve the following problems. In each case state what the letter or letters represent. *Do not solve the equations.*

a A rectangular lot is 50 feet wide and 60 feet long. If both the width and the length are increased by the same amount, the area is increased by 1200 square feet. Find the amount by which both the width and the length are increased. [5]

b At noon a train leaves New York for Buffalo. Two hours later another train leaves New York for Buffalo on a parallel track over the same route as the first. If the second train travels 28 miles per hour faster than the first and passes the first train at 5 p. m., find the rate of each train. [5]

32 A sum of money is to be distributed among 10 prize winning contestants in such a way that each one after the first receives \$10 more than the preceding person. The largest prize awarded is three times the smallest. Let a represent the smallest prize.

(1) Express the largest prize in terms of a. [1]
(2) Using the formula for the last term of an arithmetic progression, find a. [3]
(3) Find the amount distributed in prizes. [6]

33 Each of the equations in column *I* has one of the numbers in column *II* as a root. List the numbers 1–5 on your answer paper and after *each* number write one of the letters *a–g* that indicates a root of the corresponding equation. [10]

Column I	*Column II*
(1) $x^2 = 5x$	(a) 5
(2) $\dfrac{2x}{3} + \dfrac{1}{2} = \dfrac{x}{3}$	(b) 2.25
	(c) 2.125
(3) $(x^2 + 4)(4x + 5) = 0$	(d) 1.25
(4) $\sqrt{x + .125} = 1.5$	(e) -1
(5) $3^{4x-2} = 3^3$	(f) -1.25
	(g) -1.5

34 Indicate the correct completion for *each* of the following statements by listing the numbers 1–5 on your answer paper and placing after each number the letter a, b or c. [10]

(1) The graphs of the equations $x + 2y = 7$ and $x + 2y = 12$ when drawn on the same set of axes (a) intersect (b) are parallel (c) are coincident

(2) The graph of the equation $y = x^2 + 4$ is (a) symmetric with respect to the y-axis (b) symmetric with respect to the x-axis (c) tangent to the x-axis

(3) The graph of the equation $x^2 - y^2 = 16$ is (a) a circle (b) a hyperbola (c) an ellipse

(4) When drawn on the same set of axes, the graph of $y = x - 1$ will intersect the graph of $x^2 + y^2 = 25$ in (a) no point (b) one point (c) two points

(5) When drawn on the same set of axes, the graph of $y = x^2 + 7x - 10$ will intersect the graph of $y = 4x$ at the point (a) $(-5, 20)$ (b) $(5, -20)$ (c) $(2, 8)$

INTERMEDIATE ALGEBRA

Fill in the following lines:

Name of pupil..Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the prime factors of $ax^3 - a^2x^2 + ax$. 1................

2 Write the *three* factors of $a^4 - 1$. 2................

3 Find the value of $2(4)^0 - 4^{\frac{1}{2}}$ 3................

4 Write the fraction $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ as an equivalent fraction with a rational denominator. 4................

5 Express the complex fraction $\dfrac{\frac{a}{x}}{\frac{b}{x^2}}$ in simplest form. 5................

6 Solve the following pair of equations for x in terms of a and b:

$x + y = a$
$x - y = b$ 6................

7 If r varies directly as s and if $r = 13$ when $s = 52$, find s when $r = 100$. 7................

8 Write an equation of the straight line that passes through the origin and has a slope of 2. 8................

9 The parabola whose equation is $y = ax^2$ passes through the point $(2, 3)$. Find the value of a. 9................

10 Find the logarithm of 0.05728 10................

11 Find the antilogarithm of 3.4060 11................

12 The hypotenuse of a right triangle is 12 and one of its angles is 25°. Find to the *nearest tenth* the shorter leg of the triangle. 12................

13 Express the logarithm of $\dfrac{\sqrt[3]{a}}{b}$ in terms of log a and log b. 13................

Directions (14–17): The following statements refer to the equation $y = x^2 - 6x + 8$. In *each* case tell whether the statement is *true* or *false*.

14 If $y = 0$, the roots of the equation are real, unequal and rational. 14...............

15 The value of y will be positive if the value of x is greater than 4. 15...............

16 The equation of the axis of symmetry of the graph of the equation is $x = 3$. 16...............

17 The y-intercept of the graph of the equation is 8. 17...............

18 Write the third term in the expansion of $(a + b)^8$. 18...............

Directions (19–21): For *each* of the following, tell whether the statement is always true, sometimes true or never true, by writing one of the words *always, sometimes* or *never* on the line at the right.

19 If the first term of a geometric progression is a and the third term is b, then the second term is either $+\sqrt{ab}$ or $-\sqrt{ab}$. 19...............

20 In the infinite geometric progression $a + ar + ar^2 + \ldots$, if the value of r lies between 0 and 1, the sum of the terms of the progression is $\dfrac{a}{1-r}$ 20...............

21 The sum of the roots of the equation $x^2 - px + p = 0$ is equal to their product. 21...............

Directions (22–25): Indicate the correct completion for *each* of the following by writing the letter a, b or c on the line at the right.

22 The sum of the numbers $\sqrt{50}$ and $\sqrt{-50}$ is
(*a*) $10i\sqrt{2}$ (*b*) $5\sqrt{2} + 5i\sqrt{2}$ (*c*) 0 22......

23 If $\dfrac{1}{R} + \dfrac{1}{S} = \dfrac{1}{T}$, then (*a*) $T = R + S$ (*b*) $T = \dfrac{S+R}{RS}$
(*c*) $T = \dfrac{RS}{S+R}$ 23......

24 If $y = x^{-2}$ and x increases from 0, the value of y (*a*) increases (*b*) decreases (*c*) increases and then decreases 24......

25 If x pounds of salt are added to n pounds of a solution of salt and water that is 3% salt, then the total amount of salt in the resulting mixture is
(*a*) $\dfrac{x + 3n}{100}$ (*b*) $x + .03n$ (*c*) $.03\,(x + n)$ 25......

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
INTERMEDIATE ALGEBRA

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $ax(x^2 - ax + 1)$
(2) $(a^2 + 1)(a - 1)(a + 1)$
(3) 0
(4) $\dfrac{\sqrt{5} - \sqrt{2}}{3}$
(5) $\dfrac{ax}{b}$
(6) $x = \dfrac{a + b}{2}$
(7) 400
(8) $y = 2x$
(9) $\frac{3}{4}$
(10) 8.7580 — 10
(11) 2547
(12) 5.1
(13) $\frac{1}{3}\log a - \log b$
(14) true
(15) true
(16) true
(17) true
(18) $28a^6b^2$
(19) always
(20) always
(21) always
(22) *b*
(23) *c*
(24) *b*
(25) *b*