The University of the State of New York
318th High School Examination

INTERMEDIATE ALGEBRA

Wednesday, June 17, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find to the nearest tenth the roots of the equation $2x^2 - 4x - 3 = 0$ [10]

27 Given the equations $xy = 24$ and $x - 2y = 2$.
   a If the graphs of these two equations were drawn on the same set of axes, in how many points would they intersect? [2]
   b By solving algebraically the given set of equations, find the coordinates of the point (or points) of intersection of the graphs of these equations. [8]

28 Using logarithms, find to the nearest integer the value of $\frac{8.3^2 \times \sqrt{947}}{\cos 42^\circ}$ [10]

29 a Draw the graph of the equation $3x + 2y = 6$. [2]
   b On the same axes used in a, draw the graph of the equation $y = x^2 - 4x$ from $x = -1$ to $x = 5$ inclusive. [6]
   c From the graphs drawn in answer to a and b, find to the nearest tenth the coordinates of one of the points in which the two graphs intersect. [2]

*30 Find the roots of the equation $3x^3 - 4x^2 - 17x + 6 = 0$ [10]

*31 The sum of the digits of a three-digit number is 15. The sum of the hundreds digit and the tens digit is twice the units digit. The number formed by reversing the digits is 198 more than the given number.
   a Letting $h$ represent the hundreds digit, $t$ the tens digit and $u$ the units digit, write three equations that can be used to find $h$, $t$ and $u$. [3]
   b Find the values of $h$, $t$ and $u$. [6]
   c Write the number. [1]

* This question is based upon one of the optional topics in the syllabus.

[over]
Intermediate Algebra

Part III

Answer two questions from part III.

32 A light plane whose normal cruising speed is 125 miles per hour has enough fuel for 2½ hours of flying. On the outward trip it has a head wind that decreases its speed by 25 miles per hour and on the return trip a tail wind that increases its speed by 25 miles per hour. Find the greatest distance that the plane may go and have enough fuel for the return trip. \[7, 3\]

33 Write the equations that can be used to solve the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]
   a How much water must be evaporated from 32 pounds of a 4% solution of salt and water to make the result a solution that is 6% salt? \[5\]
   b A car left a certain place and traveled at a uniform rate. One hour later a second car left the same place and traveled over the same road at a rate that was 7 miles per hour faster than the rate of the first car. The second car overtook the first at a point 210 miles from the starting point. Find the rate of the second car. \[5\]

34 The owner of a motion picture theater knows that 350 tickets were sold for a certain matinee performance. The price per ticket was 70¢ for an adult and 30¢ for a child (including tax). The cashier reported $150 as the total receipts from this performance.
   a Let \(x\) represent the number of adults' tickets sold and \(y\) the number of children's tickets sold. Write two equations that can be used to find \(x\) and \(y\). \[5\]
   b Solve the set of equations written in answer to a. \[3\]
   c From the answer obtained in answer to b, what conclusion would you draw regarding the report? \[2\]

35 Each statement in column \(A\) refers to the roots of one of the equations in column \(B\). List the letters \(a-e\) on your answer paper and after each letter write the number preceding the corresponding equation. \[10\]

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>a The roots are numerically equal but opposite in sign.</td>
<td>(1) (2x^2 - 9x + 6 = 0)</td>
</tr>
<tr>
<td>b The roots are imaginary.</td>
<td>(2) (x^2 - 5x + 2 = 0)</td>
</tr>
<tr>
<td>c One of the roots is zero.</td>
<td>(3) (x^2 = 4x - 6)</td>
</tr>
<tr>
<td>d The roots are real, equal and rational.</td>
<td>(4) (x^2 - 3x^2 - 7x = 0)</td>
</tr>
<tr>
<td>e The product of the roots is 3.</td>
<td>(5) (x^2 - 2x - 3 = 0)</td>
</tr>
</tbody>
</table>

[2]
Intermediate Algebra

Fill in the following lines:

Name of pupil ............................................. Name of school ......................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Factor: \( ax + ay + a \)

2. Find the positive root of the equation \(2x^2 - 5x - 12 = 0\)

3. Solve the formula \(V = \frac{h}{6}(B + B' + 4a)\) for \(h\).

4. The smallest of three consecutive integers is \(n\). Express in terms of \(n\) the average of these integers.

5. Simplify the complex fraction \(\frac{\frac{x}{y} + \frac{y}{x}}{1} \quad \frac{1}{x^2y} \)

6. Express \(\frac{3}{\sqrt{3} - 1}\) as an equivalent fraction with a rational denominator.

7. Find the value of \(x^3 - x^9\) when \(x = 4\).

8. If \(0.000062\) is written in the form of \(6.2 \times 10^n\), what is the value of \(n\)?

9. If \(x\) varies directly as the square of \(y\) and if \(x = 100\) when \(y = 5\), find the value of \(x\) when \(y = 4\).

10. Which of the following numbers is a root of the equation \(x^2 + 4 = 0\)?
    \[(a) \quad 2 \quad (b) \quad -2 \quad (c) \quad 2i\] [Which is correct \(a\), \(b\) or \(c\)?]

11. Find the slope of the straight line which passes through the points \((0, 0)\) and \((1, 2)\).

12. Find the value of \(x\) and the value of \(y\) which satisfy the equations
    \[
    \begin{align*}
    3x + 2y &= 12 \\
    7x - 2y &= 8
    \end{align*}
    \]

13. What is the name of the graph of the equation \(x^2 - y^2 = 8\)?
    \[\boxed{3}\] [OVER]
14 Write the equation of the axis of symmetry of the graph of the equation \( y = x^2 - 4x + 5 \)

15 Write the equation of the circle whose center is the origin and which passes through the point \((0, 2)\).

16 Find the logarithm of \(0.002973\)

17 Find the antilogarithm of \(1.9067\)

18 The legs of a right triangle are 2 and 4. Find to the nearest degree the smaller acute angle of the triangle.

19 Find the sum of the infinite series \(4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} \ldots\)

20 Write the first two terms in the expansion of \((x + 2y)^5\)

21 Find the sum of the roots of the equation \(2x^2 - 8x - 5 = 0\)

22 If the discriminant of a quadratic equation with rational coefficients is 64, the roots of the equation are real, rational and unequal. Is this statement true or is it false?

23 The first term of an arithmetic progression is 8 and the fifth term is 2. Find the common difference.

24 Express the sum of the first \(n\) terms of a geometric progression whose first term is \(a\) and whose common ratio is \(r\).

25 \(\log(r^2 - s^2)\) equals \( (a) 2 \log r - 2 \log s \quad (b) \log(r + s) + \log(r - s) \quad (c) \log 2r - \log 2s \quad \) [Which is correct \(a\), \(b\) or \(c\)?]