The University of the State of New York
315TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, June 18, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Twice the square of a certain number diminished by 6 times that number is equal to —3.
   " Write an equation that can be used to find the number. [2]
   " Solve the equation written in answer to a. [Leave your answer in radical form.] [8]

27 Solve the following system of equations and check: [8, 2]
   \[ x^2 + 2y^2 = 6 \]
   \[ x - y = 1 \]

28 a Draw the graph of the equation \( y = x^2 + 4x - 3 \) from \( x = -5 \) to \( x = 1 \) inclusive. [6]
   " From the graph drawn in answer to a
      (1) estimate to the nearest tenth the roots of the equation \( x^2 + 4x - 3 = 0 \) [2]
      (2) write the equation of the axis of symmetry [1]
      (3) find a value of \( k \) for which the roots of the equation \( x^2 + 4x - 3 = k \) are imaginary [1]

29 Given the formula \( s = \sqrt{\frac{3V}{e \sin x}} \). By means of logarithms, find to the nearest tenth the value of \( s \) when \( V = 350 \), \( e = 8.30 \) and \( x = 70^\circ \). [10]

*30 Solve the equation \( 2x^3 + x^2 - 15x - 18 = 0 \). [10]

*31 In how many years \((n)\) will $645 amount to $1000 if interest is compounded annually at 5%? Use the formula \( A = P \left(1 + r\right)^n \) and give your answer to the nearest year. [10]

* This question is based upon one of the optional topics in the syllabus.

[1]

[over]
Part III

Answer one question from part III.

32. Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a. The tens digit of a two-digit number exceeds the units digit by 2. If the digits are reversed, the resulting number is 4 times the sum of the digits. Find the original number. [5]

b. It takes a motor boat 50 minutes to travel upstream a distance of 5 miles and 30 minutes to travel the same distance downstream. Find the rate of the boat in still water and the rate of the current. [5]

33. A ship was loaded with 390 tons of coal as part of its cargo. Had the rate of loading been increased by 4 tons per hour, it would have taken 2 hours less time to load the coal. How long did it actually take to load the coal? [5, 5]

Part IV

Answer one question from part IV.

34. If the blank space in each of the following statements is filled by one of the words always, sometimes or never, the resulting statement will be true. Write the numbers (1) to (5) on your answer paper and opposite each write the word that will correctly complete the corresponding statement. [Consider only cases where \( a, b \) and \( c \) are real numbers other than zero.]

(1) If \( b \) is one half the sum of \( a \) and \( c \), then \( a, b \) and \( c \) are ... in arithmetic progression. [2]

(2) The graph of the equation \( ax^2 + bx + c = y \) ... passes through the origin. [2]

(3) If \( a \) and \( c \) are opposite in sign, then the roots of \( ax^2 + bx + c = 0 \) are ... imaginary. [2]

(4) If \( c \) is added to both the numerator and the denominator of fraction \( \frac{a}{b} \), \( a, b \) and \( c \) being positive integers, then the value of the fraction is ... increased. [2]

(5) If \( \log a + \log c = 2 \log b \), then \( a, b \) and \( c \) are ... in geometric progression. [2]

35. A mixture of \( m \) pounds of sand and cement is \( c \) per cent cement.

a. Express the number of pounds of cement in the mixture in terms of \( c \) and \( m \). [3]

b. If \( x \) pounds of sand are added to this mixture, it will then be \( d \) per cent cement. Express the number of pounds of cement in the new mixture in terms of \( d, m \) and \( x \). [3]

c. Find \( x \) in terms of \( c, d \) and \( m \). [4]
Name of pupil..........................................................Name of school..........................................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the positive root of the equation $y^2 + 5y - 6 = 0$. 1.

2 Express $2\sqrt{-9}$ in terms of $i$. 2.

3 Express $\frac{3}{\sqrt{3} - 1}$ as an equivalent fraction with a rational denominator. 3.

4 Solve for $x$: $2\sqrt{x - 1} - 3 = 5$. 4.

5 Solve for $S$: $A = \frac{R - S}{T}$. 5.

6 Reduce to lowest terms the fraction $\frac{1 - a^2}{a + 1}$. 6.

7 If $y = 2x^2 + x^{-4}$, find $y$ when $x = 3$. 7.

8 The distance between $A$ and $B$ on a straight road is 1000 ft. If the road inclines 7° to the horizontal, find to the nearest foot the difference in altitude between $A$ and $B$. 8.

9 Find log 3683 9.

10 Find the number whose logarithm is 0.4343 10.

11 If $a = \frac{v^2}{r}$, express log $a$ in terms of log $v$ and log $r$. 11.

12 Write an equation of the straight line whose slope is 2 and whose $y$-intercept is 5. 12.

13 Write an equation expressing the relationship between $x$ and $y$ shown by the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-1</td>
<td>1</td>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

14 If $x$ varies inversely as $y$ and if $x = 2$ when $y = 3$, find $x$ when $y = 6$. 14. [OVER]

[OVER]
15. The first term of an arithmetic progression is $-3$, the last term is 56, and the number of terms is 20. Find the sum.

16. The first term of a geometric progression is $\frac{1}{10}$ and the common ratio is 2. Find the 10th term.

17. Find the sum of the infinite series $2, \frac{4}{3}, \frac{8}{9}, \ldots$

18. Find the sum of the roots of the equation $2x^2 - 5x - 4 = 0$

19. Find the product of the roots of the equation $x^2 + 3x - 8 = 0$

20. Simplify the complex fraction $\frac{\frac{1}{r} + \frac{1}{s}}{\frac{1}{rs}}$

21. The graph of the equation $y = x^2 + 3x + k$ passes through the point $(2, 0)$. Find $k$.

Directions (22–25): Indicate the correct completion for each of the following by writing on the line at the right the letter $a$, $b$ or $c$.

22. The expression $(x^4)^2$ is equal to
(a) $x^8$  (b) $x^8$  (c) $x^8$  22.

23. The graph of the equation $x^2 + \frac{y^2}{2} = 18$ is
(a) a circle  (b) an ellipse
(c) a hyperbola  23.

24. When drawn on the same axes, the graphs of the equations $xy = -6$ and $x = y$ (a) intersect in 2 points  (b) intersect in only 1 point  (c) do not intersect  24.

25. If the discriminant of the equation $ax^2 + bx + c = 0$ is 4, the graph of the equation $y = ax^2 + bx + c$ (a) intersects the $x$-axis in two distinct points (b) is tangent to the $x$-axis  (c) has no point in common with the $x$-axis  25.