The University of the State of New York
309th High School Examination

INTERMEDIATE ALGEBRA

Wednesday, June 21, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II
Answer three questions from part II.

26 Find, to the nearest tenth, the roots of the equation \(3x^2 - 7x = 2\). [10]

27 Solve the following system of equations, group your answers, and check one set: \([7, 2, 1]\)

\[
x^2 + 4y^2 = 13
\]

\[
x - 2y = 5
\]

28 Given the formula \(T = \pi \sqrt{\frac{I}{Mgh}}\). Using logarithms, find \(T\) to the nearest hundredth when \(I = 53,400\), \(M = 278\), \(g = 980\) and \(h = 4.3\). [Use \(\pi = 3.14\).] [10]

29 \(a\) Draw the graph of \(y = x^2 - 2x - 8\) from \(x = -3\) to \(x = +5\). [7]

\(b\) Write the equation of the axis of symmetry. [2]

\(c\) Find the minimum value of \(x^2 - 2x - 8\). [1]

*30 In how many years \((n)\) will \$500 amount to \$1000 if interest is compounded annually at 4\%? Use the formula \(A = P(1 + r)^n\) and give your answer to the nearest year. [19]

*31 Find the three roots of the equation \(2x^3 - 3x^2 - 11x + 6 = 0\). [10]

* This question is based upon one of the optional topics in the syllabus.
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Part III
Answer one question from part III.

32 How many pounds of pure water should be evaporated from 70 pounds of salt water, 4% of which (by weight) is pure salt, to increase it to a 5% solution? [7, 3]

33 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a A man invested $6000 in two enterprises. At the end of the first year he found that he had gained 6% on one of the sums invested and had lost 4% on the other. His net profit for the year was $160. How much did he invest at each rate? [5]

b Three numbers are in the ratio 1:2:5. If 3 is subtracted from the first number, the second is left unchanged and 9 is added to the third, these three numbers taken in the same order then form a geometric progression. Find the numbers. [5]

Part IV
Answer one question from part IV.

34 For each of the following statements, in which $a$, $b$ and $c$ are real numbers, indicate whether the information given is too little, just enough, or more than is necessary, to justify the conclusion.

(1) If the graph of $y = mx + b$ is parallel to a line whose equation is given, then the value of $m$ and the value of $b$ are determined. [2]

(2) If the center of a circle is at the origin and the circle passes through the point $(a, b)$, then the radius of the circle is $\sqrt{a^2 + b^2}$. [2]

(3) If the graph of $ax^2 + by^2 = c$ is an ellipse, then $a$, $b$ and $c$ have the same sign. [2]

(4) If, in the equation $y = ax^2 + bx + c$, $a$ and $c$ are opposite in sign, then the graph of the equation intersects the $x$-axis. [2]

(5) If, in the equation $ax^2 - by^2 = c$, $a$, $b$ and $c$ are positive and $a$ is not equal to $b$, then the graph of the equation is a hyperbola. [2]

35 A group of men agreed to contribute equally toward purchasing a gift which was to cost $c$ dollars. Later $n$ men joined the group, thus causing the individual contribution to be $d$ dollars less. These facts are represented graphically in the accompanying figure. OR represents the number of men in the group originally, OS the number after $n$ joined the group, and RT (equal to SV) the cost of the gift.

Let OR be represented by $x$.

a Express OS in terms of $x$ and $n$. [1]

b Express the slope of line OT in terms of $c$ and $x$. [2]

c Express the slope of line OV in terms of $c$, $x$ and $n$. [2]

d Write an equation that would be used to find $x$ in terms of $c$, $d$ and $n$. [5]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Reduce to lowest terms: \( \frac{x - 2}{x^2 - 4} \) 1..............

2 Express in terms of \( i \) the sum of \( \sqrt{-16} \) and \( \sqrt{-9} \). 2..............

3 Write the fraction \( \frac{1}{\sqrt{3} - 1} \) with a rational denominator. 3..............

4 Simplify the complex fraction \( \frac{1}{x} + \frac{1}{y} \) \( \frac{1}{xy} \) 4..............

5 Using the formula \( A = P(1 + rt) \), find \( A \) when \( P = 500 \), \( r = .03 \) and \( t = 15 \). 5..............

6 If \( r \) varies directly as \( s \) and if \( r = 3 \) when \( s = 8 \), find \( r \) when \( s = 12 \). 6..............

7 Solve for \( x \) the equation \( \sqrt{x + 2} = 3 \) 7..............

8 Solve the following system of equations for \( x \) and \( y \):
   \[ y - x = 4 \]
   \[ y - 2x = 0 \]

9 Find the positive root of the equation \( 2x^2 - 3x - 2 = 0 \). 9..............

10 Find the sum of the roots of the equation \( x^2 - 3x = k \). 10..............

11 Find the product of the roots of the equation \( x^2 - 3x = 0 \). 11..............

12 Solve the formula \( T = 2\pi r^2 + 2\pi rh \) for \( h \). 12..............

13 Find the logarithm of 8.324. 13..............

14 Find the number whose logarithm is 9.4356 - 10. 14..............

15 The hypotenuse of a right triangle is 12 and one of the acute angles is \( 28^\circ \). Find, to the nearest tenth, the longer leg of the triangle. 15..............

16 The first term of an arithmetic progression is \( \frac{1}{3} \) and the sixth term is 20. Find the common difference. 16..............

17 Find the sum of the infinite geometric progression whose first term is 3 and whose common ratio is \( \frac{1}{4} \). 17..............

18 Write the first two terms of the expansion of \( (x + y)^3 \). 18..............

19 Write the linear equation expressing the relationship between \( x \) and \( y \) shown in the following table:

| \( x \) | -1 | 0 | 2 | 6 |
| \( y \) | -5 | -2 | 4 | 16 |

[3] 19..............

[over]
Directions (questions 20-22) — Indicate whether each of the following statements is true or false by writing the word true or the word false on the line at the right. [In questions 20-21, \( x \) and \( y \) are real and not equal to zero.]

20 The expression \( x^2 - y^6 \) is equal to \( \frac{1}{x^2} - 1 \). 20. .................

21 The expression \( \sqrt{x} + \sqrt{y} \) is equal to \( \sqrt{x + y} \). 21. .................

22 If \( n \) is a positive odd integer, \( \sqrt{-1} \) is an imaginary number. 22. .................

Directions (questions 23-25) — Indicate the correct answer to each of the following by writing the letter \( a \), \( b \) or \( c \) on the line at the right.

23 The roots of the equation \( 2x^2 - 8x + 3 = 0 \) are (a) equal and rational (b) unequal and rational (c) unequal and irrational 23. .......

24 The straight line \( y = mx \) and the circle \( x^2 + y^2 = 9 \), when drawn on the same set of axes, (a) intersect regardless of what value \( m \) may have (b) may be tangent (c) may not intersect 24. .......

25 If \( \log r + \log s = \log t \), then (a) \( \log (r + s) = \log t \) (b) \( r + s = t \) (c) \( rs = t \) 25. .......

[4]